Extra credit #1

EE 194: Advanced VLSI
Spring 2018

This extra-credit problem will be due the same time as the regular homework, as indicated on the class calendar. You can either write up your answers and turn this in via *provide*, or just stop by during office hours and explain your solutions in person (without needing to write them down).

Like most(all) of the extra-credit problems, the answers to these questions require material that is not covered in the class, and usually is not in the textbook either.

We mentioned in class that, most of the time, the energy expended in a series resistor is the same as the energy stored or discharged in a capacitor. This is our *energy hypothesis*. Now let's prove when it does (and doesn't) work.

1. (easy) Consider a capacitor *C* initially charged to a voltage *V*. Discharge it by connecting a resistor *R* across its two terminals. Prove that the energy lost by the capacitor all becomes heat energy created by the resistor. Prove that this is true at any point during the discharge process, even before the capacitor is fully discharged. Prove that this energy is independent of *R*.

2. (medium) Now charge *C* via a series circuit: a voltage source *V* through a resistor *R* and then *C*. Prove that when we fully charge *C* from 0 volts to *V* volts, the hypothesis is true. Prove that this holds even if *R* changes while the capacitor is charging (which is good, since an MOS transistor in its active mode can be modeled as a variable resistor). Hint: this problem does take a small bit of calculus, but does not require solving any differential equations.
3. (easy, given #2) Prove that if we only partially charge *C* (i.e., not all the way to *V* volts), then the hypothesis is false. Does the charging process waste more energy near the beginning or near the end? There is a style of circuit design called *adiabatic logic* that takes advantage of this.

4. (easy) Now replace the voltage source *V* with a constant-current source *I*. Prove that, for this circuit, the hypothesis is no longer true when charging the capacitor.

5. (might take some thought) Given that an MOS transistor in saturation is often modeled as a constant-current source, and that we just showed that our energy hypothesis does *not* work for a constant-current source, why do we often claim that our hypothesis works well in integrated circuits?