Joint Typicality Analysis for Half-Duplex Cooperative Communication

Ahmad Abu Al Haija, and Mai Vu,
Department of Electrical and Computer Engineering
McGill University
Montreal, QC H3A 2A7
Emails: ahmad.abualhaija@mail.mcgill.ca, mai.h.vu@mcgill.ca

***Abstract*—We propose a half-duplex cooperative scheme for a discrete memoryless channel (DMC) consisting of two users communicating with one destination. The half-duplex constraint is satisfied by performing the communication over 3 time slots with variable durations in each code block. Each user alternatively transmits and receives during the first 2 time slots, then both of them transmit during the last one. Different from [1], here we use joint typicality instead of maximum likelihood (ML) decoding to derive the rate region. The main contribution is in the joint decoding and proof techniques that concurrently combine code segments of different lengths.**

I. Introduction

Recently, a continuous growth in multimedia services demands new technologies that meet the high speed and throughput requirements. Cooperative communications offer a viable solution. Examples include the multiple access channel (MAC) with generalized feedback and the relay channel. Achievable rate region for MAC with generalized feedback has been derived in [2] using block Markov encoding [3] and backward decoding [4]. The application of this coding scheme into cellular networks operating over fading channels has been analyzed in [5]. The relay channel is another model introduced in [6]. The capacity region for the degraded relay channel has been derived in [7] along with several schemes such as decode-forward and compress-forward. These coding schemes assume full-duplex operation for each user.

On the other hand, half duplex cooperative communications have received a great attention of research recently as a more practice constraint [8]–[12]. The capacity region of the relay channel with orthogonal components from the sender to the relay and from the sender and relay to the destination, which models frequency division, has been derived in [13].

In [1], we proposed a new cooperative half-duplex scheme that combines the ideas of the MAC with generalized feedback and relay channels with partial decode-forward relaying. The proposed scheme consists of 2 users in half-duplex mode cooperating in order to increase their rates to the destination. Each transmission is independent and is divided into 3 time slots. These time slots implement the half-duplex requirement while the independent blocks remove the Markovity among the transmitted codewords and the need for backward decoding as used in [2], [5]. These characteristics allow the scheme to meet practical half-duplex and delay constraints. The achievable rate region was derived in [1] using superposition encoding and joint (ML) decoding. In this paper, we provide another proof of the achievable rate region using joint typicality decoding [14] where the destination will jointly decode what it receives during the three time slots in each transmitted block. The proof involves an analysis of error events defined jointly over multiple time slots with different lengths. As a consequence, the probability of these error events becomes the product of the error probability in each time slot. Then, the rate constraints can be obtained by combining the upper bounds of these error probabilities.

The remainder of this paper is organized as follows. The channel model is described in Section II. Section III explains the encoding, and the decoding schemes. Section IV then provides the achievable rate region, the joint typicality error analysis, and some numerical results. Finally, Section V concludes the paper.

II. Channel Model

The two user discrete memoryless cooperative half-duplex channel can be defined as shown in Fig.1. It consists of two input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$, three output alphabets $\mathcal{Y}$, $\mathcal{Y}_{12}$, and $\mathcal{Y}_{21}$, and three conditional transition probabilities $p(y|x_1,x_2)$, $p(y,y_{12}|x_1)$, and $p(y,y_{21}|x_2)$, respectively. This channel is similar to that described in [2]. However, in order to satisfy the half duplex constraint, we require that no two transition probabilities occur simultaneously. Because of this new requirement, the coding scheme given in [2] can not be directly applied to derive the rate region.

A $(n,\lceil 2^{nR_1}\rceil,\lceil 2^{nR_2}\rceil,P_e)$ code for this channel consists of two message sets $W_1=\{1,\ldots,\lceil 2^{nR_1}\rceil\}$, and $W_2=\{1,\ldots,\lceil 2^{nR_2}\rceil\}$, two encoding functions $f_{1i},f_{2i},\quad i=1,\ldots,n$, and one decoding function g:

$$\begin{aligned}
&f_{1i}: W_1\times\mathcal{Y}_{21}^{i-1}\rightarrow\mathcal{X}_1, i=1,\ldots,n\\
&f_{2i}: W_2\times\mathcal{Y}_{12}^{i-1}\rightarrow\mathcal{X}_2, i=1,\ldots,n\\
&g:\mathcal{Y}^n\rightarrow W_1\times W_2
\end{aligned}\tag{1}$$

Finally, P_e is the average error probability defined as the $P_e=P(g(Y^n)\neq(W_1,W_2))$. A rate pair (R_1,R_2) is said to be achievable if there exists a $(n,\lceil 2^{nR_1}\rceil,\lceil 2^{nR_2}\rceil,P_e)$ code such that $P_e\rightarrow 0$ as $n\rightarrow\infty$. The closure of the set of all achievable rates (R_1,R_2) is the capacity region of the half duplex cooperative scheme.

III. A Half-Duplex Cooperative Scheme

In [1], we proposed a half duplex cooperative scheme in the following manner. Each of the independent transmitted blocks

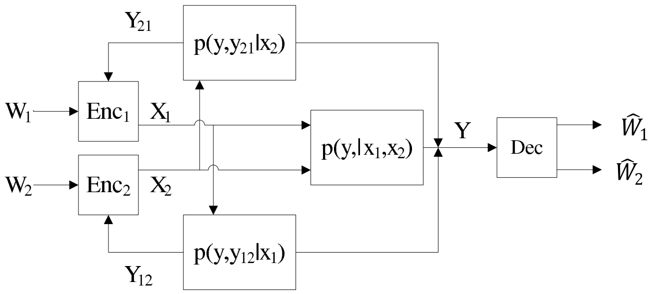

Fig. 1. Channel model for the half duplex cooperative scheme.

is divided into 3 time slots with variable durations. The first user will transmit during the first time slot and receive during the second time slot while the second user does the opposite. Then in the third time slot, both of them will cooperatively transmit. Finally, the destination will jointly decode what it receives during the 3 time slots. The encoding is performed by employing rate splitting and superposition coding while the decoding is performed using joint (ML) decoding.

In this paper, we will use joint typicality decoding to derive the achievable rate region. The transmission from the first user is performed as follows. First, it divides its message, W_1, into three parts. Then, the first and the third parts W_{10}, and W_{13} are private and are transmitted directly to the destination at rates R_{10}, and R_{13}, respectively. The second part W_{12} is public and is transmitted to the destination in cooperation with the second user at rate R_{12}. Similar transmission is performed by the second user. Next, we describe formally the encoding and decoding schemes.

A. Encoding Scheme

This encoding scheme is similar to [1]. We will rewrite it here for the ease of reference. Denote P^* as the following joint distribution:

$$P^* = p(u)p(v)p(x_{10}|u)p(x_{13}|u,v)p(x_{20}|v)p(x_{23}|u,v). \quad (2)$$

1) Codebook generation: Fix P^* and then generate:

- $|W_{12}| = \lceil 2^{nR_{12}} \rceil$ sequences $u^n(w_{12})$ iid $\sim \prod_{i=1}^{n} p(u_i)$.
- $|W_{21}| = \lceil 2^{nR_{21}} \rceil$ sequences $v^n(w_{21})$ iid $\sim \prod_{i=1}^{n} p(v_i)$.

Then for each $u^n(w_{12})$, and $v^n(w_{21})$, generate:

- $|W_{10}| = \lceil 2^{nR_{10}} \rceil$ sequences $x_{10}^n(w_{10}, w_{12})$ iid $\sim \prod_{i=1}^{n} p(x_{10i}|u_i)$, and
- $|W_{20}| = \lceil 2^{nR_{20}} \rceil$ sequences $x_{20}^n(w_{20}, w_{21})$ iid $\sim \prod_{i=1}^{n} p(x_{20i}|v_i)$, respectively.

Finally, for each pair $(u^n(w_{12}), v^n(w_{21}))$, generate:

- $|W_{13}| = \lceil 2^{nR_{13}} \rceil$ sequences $x_{13}^n(w_{13}, w_{12}, w_{21})$ iid $\sim \prod_{i=1}^{n} p(x_{13i}|u_i, v_i)$
- $|W_{23}| = \lceil 2^{nR_{23}} \rceil$ sequences $x_{23}^n(w_{23}, w_{12}, w_{21})$ iid $\sim \prod_{i=1}^{n} p(x_{23i}|u_i, v_i)$.

The encoding and decoding at each block can be explained with the help of Table I, where $0 \leq \alpha_1 + \alpha_2 \leq 1$.

2) Encoding: In order to send the message pair (W_1, W_2), the first user sends $x_{10}^{\alpha_1 n}(w_{10}, w_{12})$ during the first $\alpha_1 n$ uses of the channel. At the ends of the 2nd time slots, this user will have the estimated values $(\tilde{w}_{20}, \tilde{w}_{21})$. Then, during the last $(1-\alpha_1-\alpha_2)n$ uses of the channel, the first user sends $x_{13,(\alpha_1+\alpha_2)n+1}^{n}(w_{13}, w_{12}, \tilde{w}_{21})$. The transmission of W_2 from the second user is done in a similar way. Hence, we can easily see the partial decode-forward cooperation in the sense that during the first two time slots, each user decodes both private and public parts of the message of its partner. Then, each forwards only the public part in addition to its second private message during the 3rd time slot.

B. Decoding Scheme

In this section, we will use joint typicality decoding instead of the (ML) decoding as used in [1]. We will also see that both decoding techniques lead to the same achievable region, but the proofs are different and can not be trivially inferred from one another. Thus in this paper, we analyze in detail the joint typicality decoding.

1) Decoding at each user: At the end of the 1st time slot, the second user uses joint typicality decoding rule to decode (w_{10}, w_{12}) from its received sequence $\boldsymbol{Y}_{12}$. The second user decodes $(\tilde{w}_{10}, \tilde{w}_{12})$ as the transmitted symbols if it is the unique message pair such that

$$(u^{\alpha_1 n}(\tilde{w}_{12}), x_{10}^{\alpha_1 n}(\tilde{w}_{10}, \tilde{w}_{12}), \boldsymbol{Y}_{12}) \in A_\epsilon^{\alpha_1 n}.$$

Similarly, the first user decodes the unique message pair $(\tilde{w}_{20}, \tilde{w}_{21})$ such that

$$(v^{\alpha_2 n}(\tilde{w}_{21}), x_{20}^{\alpha_2 n}(\tilde{w}_{20}, \tilde{w}_{21}), \boldsymbol{Y}_{21}) \in A_\epsilon^{\alpha_2 n}.$$

2) Decoding at the destination: At the destination, since our transmitted blocks are independent, we can decode the message transmitted in each block at the end of that block. The destination performs joint typicality decoding using the received signals during all 3 time slots in each block. Hence, the destination looks for a unique $(\hat{w}_{12}, \hat{w}_{21}, \hat{w}_{10}, \hat{w}_{20}, \hat{w}_{13}, \hat{w}_{23})$ such that (3) is satisfied.

IV. Achievable Rate Region Based on Joint Typicality

A. Achievable Rate Region

With the scheme described in Section III, the achievable rate region is the convex closure of the rate-tuples $(R_{10}, R_{12}, R_{13}, R_{20}, R_{21}, R_{23})$ satisfying:

$$\begin{aligned}
R_{10} &\leq \min\left(\alpha_1 I(X_{10};Y_1|U), \alpha_1 I(X_{10};Y_{12}|U)\right) = I_1 \\
R_{10} + R_{12} &\leq \alpha_1 I(X_{10};Y_{12}) = I_2 \\
R_{20} &\leq \min\left(\alpha_2 I(X_{20};Y_2|V), \alpha_2 I(X_{20};Y_{21}|V)\right) = I_3 \\
R_{20} + R_{21} &\leq \alpha_2 I(X_{20};Y_{21}) = I_4 \\
R_{13} &\leq (1-\alpha_1-\alpha_2) I(X_{13};Y_3|U,V,X_{23}) = I_5 \\
R_{23} &\leq (1-\alpha_1-\alpha_2) I(X_{23};Y_3|U,V,X_{13}) = I_6 \\
R_{13} + R_{23} &\leq (1-\alpha_1-\alpha_2) I(X_{13},X_{23};Y_3|U,V) = I_7 \\
R_1 + R_{23} &\leq \alpha_1 I(X_{10};Y_1) + \\
&\qquad (1-\alpha_1-\alpha_2) I(X_{13},X_{23};Y_3|V) = I_8 \\
R_2 + R_{13} &\leq \alpha_2 I(X_{20};Y_2) + \\
&\qquad (1-\alpha_1-\alpha_2) I(X_{13},X_{23};Y_3|U) = I_9 \\
R_1 + R_2 &\leq \alpha_1 I(X_{10};Y_1) + \alpha_2 I(X_{20};Y_2) + \\
&\qquad (1-\alpha_1-\alpha_2) I(X_{13},X_{23};Y_3) = I_{10}
\end{aligned} \quad (4)$$

for some P^* as defined in (2) and $0 \leq \alpha_1 + \alpha_2 \leq 1$. Now, by applying Fourier-Motzkin Elimination (FME) to the

	1st slot with length $\alpha_1 n$	2nd slot with length $\alpha_2 n$	3rd slot with length $(1-\alpha_1-\alpha_2)n$
First user	$x_{10}^{\alpha_1 n}(w_{10}, w_{12})$	$--$	$x_{13,(\alpha_1+\alpha_2)n+1}^{n}(w_{13}, w_{12}, \tilde{w}_{21})$
Second user	$--$	$x_{20}^{\alpha_2 n}(w_{20}, w_{12})$	$x_{23,(\alpha_1+\alpha_2)n+1}^{n}(w_{23}, \tilde{w}_{12}, w_{21})$
Y_{21}	$--$	$(\tilde{w}_{20}, \tilde{w}_{21})$	$--$
Y_{12}	$(\tilde{w}_{10}, \tilde{w}_{12})$	$--$	$--$
Y	$\boldsymbol{Y}_1$	$\boldsymbol{Y}_2$	$\boldsymbol{Y}_3$
	$(\hat{w}_{12}, \hat{w}_{21}, \hat{w}_{10}, \hat{w}_{20}, \hat{w}_{13}, \hat{w}_{23})$		

Table I: The encoding and decoding schemes for half duplex cooperative scheme

$$\begin{aligned}&\{(u^{\alpha_1 n}(\hat{w}_{12}), x_{10}^{\alpha_1 n}(\hat{w}_{10}, \hat{w}_{12}), \boldsymbol{Y}_1) \in A_\epsilon^{\alpha_1 n}, \text{ and } (v^{\alpha_2 n}(\hat{w}_{21}), x_{20}^{\alpha_2 n}(\hat{w}_{20}, \hat{w}_{21}), \boldsymbol{Y}_2) \in A_\epsilon^{\alpha_2 n}, \text{and} \\ &(u^{(1-\alpha_1-\alpha_2)n}(\hat{w}_{12}), v^{(1-\alpha_1-\alpha_2)n}(\hat{w}_{21}), x_{13}^{(1-\alpha_1-\alpha_2)n}(\hat{w}_{13}, \hat{w}_{12}, \hat{w}_{21}), x_{23}^{(1-\alpha_1-\alpha_2)n}(\hat{w}_{23}, \hat{w}_{12}, \hat{w}_{21}), \boldsymbol{Y}_3) \in A_\epsilon^{(1-\alpha_1-\alpha_2)n}\}.\end{aligned} \tag{3}$$

$$\begin{aligned}
E_1 :=&\{(u^{\alpha_1 n}(1), x_{10}^{\alpha_1 n}(1,1), \boldsymbol{Y}_1) \notin A_\epsilon^{\alpha_1 n}\};\\
E_2 :=&\{(u^{\alpha_1 n}(1), x_{10}^{\alpha_1 n}(w_{10},1), \boldsymbol{Y}_1) \in A_\epsilon^{\alpha_1 n} \text{ for some } w_{10} \neq 1\};\\
E_3 :=&\{(v^{\alpha_2 n}(1), x_{20}^{\alpha_2 n}(1,1), \boldsymbol{Y}_2) \notin A_\epsilon^{\alpha_2 n}\};\\
E_4 :=&\{(v^{\alpha_2 n}(1), x_{20}^{\alpha_2 n}(w_{20},1), \boldsymbol{Y}_2) \in A_\epsilon^{\alpha_2 n} \text{ for some } w_{20} \neq 1\};\\
E_5 :=&\{(u^{(1-\alpha_1-\alpha_2)n}(1), v^{(1-\alpha_1-\alpha_2)n}(1), x_{13}^{(1-\alpha_1-\alpha_2)n}(1,1,1), x_{23}^{(1-\alpha_1-\alpha_2)n}(1,1,1), \boldsymbol{Y}_3) \notin A_\epsilon^{(1-\alpha_1-\alpha_2)n}\};\\
E_6 :=&\{(u^{(1-\alpha_1-\alpha_2)n}(1), v^{(1-\alpha_1-\alpha_2)n}(1), x_{13}^{(1-\alpha_1-\alpha_2)n}(w_{13},1,1), x_{23}^{(1-\alpha_1-\alpha_2)n}(1,1,1), \boldsymbol{Y}_3) \in A_\epsilon^{(1-\alpha_1-\alpha_2)n} \text{ for some } w_{13} \neq 1\};\\
E_7 :=&\{(u^{(1-\alpha_1-\alpha_2)n}(1), v^{(1-\alpha_1-\alpha_2)n}(1), x_{13}^{(1-\alpha_1-\alpha_2)n}(1,1,1), x_{23}^{(1-\alpha_1-\alpha_2)n}(w_{23},1,1), \boldsymbol{Y}_3) \in A_\epsilon^{(1-\alpha_1-\alpha_2)n} \text{ for some } w_{23} \neq 1\};\\
E_8 :=&\{(u^{(1-\alpha_1-\alpha_2)n}(1), v^{(1-\alpha_1-\alpha_2)n}(1), x_{13}^{(1-\alpha_1-\alpha_2)n}(w_{13},1,1), x_{23}^{(1-\alpha_1-\alpha_2)n}(w_{23},1,1), \boldsymbol{Y}_3) \in A_\epsilon^{(1-\alpha_1-\alpha_2)n} \text{ for some}\\
&(w_{13} \neq 1, w_{23} \neq 1)\};\\
E_9 :=&\{(u^{\alpha_1 n}(1), x_{10}^{\alpha_1 n}(1,1), \boldsymbol{Y}_1) \notin A_\epsilon^{\alpha_1 n}, \text{ and } (v^{\alpha_2 n}(1), x_{20}^{\alpha_2 n}(1,1), \boldsymbol{Y}_2) \notin A_\epsilon^{\alpha_2 n}, \text{and}\\
&(u^{(1-\alpha_1-\alpha_2)n}(1), v^{(1-\alpha_1-\alpha_2)n}(1), x_{13}^{(1-\alpha_1-\alpha_2)n}(1,1,1), x_{23}^{(1-\alpha_1-\alpha_2)n}(1,1,1), \boldsymbol{Y}_3) \notin A_\epsilon^{(1-\alpha_1-\alpha_2)n}\};\\
E_{10} :=&\{(u^{\alpha_1 n}(w_{12}), x_{10}^{\alpha_1 n}(w_{10}, w_{12}), \boldsymbol{Y}_1) \in A_\epsilon^{\alpha_1 n}, \text{and}(v^{\alpha_2 n}(1), x_{20}^{\alpha_2 n}(1,1), \boldsymbol{Y}_2) \in A_\epsilon^{\alpha_2 n}, \text{and } (u^{(1-\alpha_1-\alpha_2)n}(w_{12}), v^{(1-\alpha_1-\alpha_2)n}(1),\\
&x_{13}^{(1-\alpha_1-\alpha_2)n}(w_{13}, w_{12}, 1), x_{23}^{(1-\alpha_1-\alpha_2)n}(w_{23}, w_{12}, 1), \boldsymbol{Y}_3) \in A_\epsilon^{(1-\alpha_1-\alpha_2)n} \text{ for some } w_{12} \neq 1 \text{ and any } (w_{10}, w_{13}, w_{23})\};\\
E_{11} :=&\{(u^{\alpha_1 n}(1), x_{10}^{\alpha_1 n}(1,1), \boldsymbol{Y}_1) \in A_\epsilon^{\alpha_1 n}, \text{and}(v^{\alpha_2 n}(w_{21}), x_{20}^{\alpha_2 n}(w_{20}, w_{21}), \boldsymbol{Y}_2) \in A_\epsilon^{\alpha_2 n}, \text{and}(u^{(1-\alpha_1-\alpha_2)n}(1), v^{(1-\alpha_1-\alpha_2)n}(w_{21}),\\
&x_{13}^{(1-\alpha_1-\alpha_2)n}(w_{13}, 1, w_{21}), x_{23}^{(1-\alpha_1-\alpha_2)n}(w_{23}, 1, w_{21}), \boldsymbol{Y}_3) \in A_\epsilon^{(1-\alpha_1-\alpha_2)n} \text{ for some } w_{21} \neq 1 \text{ and any } (w_{20}, w_{13}, w_{23})\};\\
E_{12} :=&\{(u^{\alpha_1 n}(w_{12}), x_{10}^{\alpha_1 n}(w_{10}, w_{12}), \boldsymbol{Y}_1) \in A_\epsilon^{\alpha_1 n}, \text{ and } (v^{\alpha_2 n}(w_{21}), x_{20}^{\alpha_2 n}(w_{20}, w_{21}), \boldsymbol{Y}_2) \in A_\epsilon^{\alpha_2 n}, \text{ and}\\
&(u^{(1-\alpha_1-\alpha_2)n}(w_{12}), v^{(1-\alpha_1-\alpha_2)n}(w_{21}), x_{13}^{(1-\alpha_1-\alpha_2)n}(w_{13}, w_{12}, w_{21}), x_{23}^{(1-\alpha_1-\alpha_2)n}(w_{23}, w_{12}, w_{21}), \boldsymbol{Y}_3) \in A_\epsilon^{(1-\alpha_1-\alpha_2)n}\\
&\text{for some } (w_{12} \neq 1, w_{21} \neq 1) \text{ and any } (w_{10}, w_{13}, w_{20}, w_{23})\}.
\end{aligned} \tag{6}$$

inequalities in (4), the achievable rates in terms of $R_1 = R_{10}+R_{12}+R_{13}$ and $R_2 = R_{20}+R_{21}+R_{23}$ can be expressed as

$$\begin{aligned}
R_1 &\leq I_2 + I_5\\
R_2 &\leq I_4 + I_6\\
R_1 + R_2 &\leq I_7 + I_2 + I_4\\
R_1 + R_2 &\leq I_2 + I_9\\
R_1 + R_2 &\leq I_4 + I_8\\
R_1 + R_2 &\leq I_{10}.
\end{aligned} \tag{5}$$

B. Joint Typicality Error Analysis

Without loss of generality, assume that the message vector $(w_{12} = w_{21} = w_{10} = w_{20} = w_{13} = w_{23} = 1)$ was sent. The error events at each user can be analyzed as that for the broadcast channel given in [14]. To make these error probabilities approach zero, the rate constraints (I_2, I_4) and the second part of (I_1, I_3) must be satisfied.

On the other hand, decoding error occurs at the destination only if one of the events in (6) occurs.

By using LLN and the packing lemma [14], it can be shown that the probabilities of the above errors go to zero as $n \to \infty$ if the rate constraints involving $(I_5 - I_{10})$ and the first part of (I_1, I_3) are satisfied as given in (4). Because of space limitation, we provide here a full analysis for E_{12} only since it is the most complicated event. $P_{E_{12}}$ can be expressed as

$$P_{E_{12}} = \sum_{i=1}^{2^{n(R_1+R_2)}-1} P_{E_{12i}},$$

where $P_{E_{12i}}$ is the probability of error for a particular set of messages $(w_{10}, w_{12}, w_{13}, w_{20}, w_{21}, w_{23})$. Since this error event is defined jointly over all 3 time slots, its probability can be expressed as a product

$$P_{E_{12i}} = P_{E^1_{12i}} \times P_{E^2_{12i}} \times P_{E^3_{12i}},$$

where $(E^1_{12i}, E^2_{12i}, E^3_{12i})$ are the error events correspond to the first, second, and third time slot, respectively. $P_{E^1_{12i}}$ can be expressed as

$$
\begin{aligned}
P_{E^1_{12i}} = & \sum_{(u,x_{10},y_1)\in A_\epsilon^{\alpha_1 n}} p(u)p(x_{10}|u)p(y_1) \\
\leq & 2^{\alpha_1 n(H(U,X_{10},Y_1)+\epsilon)} \\
& \cdot 2^{-\alpha_1 n(H(U,X_{10})-\epsilon)} \cdot 2^{-\alpha_1 n(H(Y_1)-\epsilon)} \\
= & 2^{-\alpha_1 n(I(U,X_{10};Y_1)-3\epsilon)} \\
= & 2^{-\alpha_1 n(I(X_{10};Y_1)-3\epsilon)}.
\end{aligned}
$$

Similarly, $P_{E^2_{12i}} \leq 2^{-\alpha_2 n(I(X_{20};Y_2)-3\epsilon)}$.
Finally, $P_{E^3_{12i}}$ can be expressed as

$$
\begin{aligned}
P_{E^3_{12i}} = & \sum_{(u,v,x_{13},x_{23},y_3)\in A_\epsilon^{(1-\alpha_1-\alpha_2)n}} \\
& p(u)p(v)p(x_{13}|u,v)p(x_{23}|u,v)p(y_3) \\
\leq & 2^{(1-\alpha_1-\alpha_2)n(H(U,V,X_{13},X_{23},Y_3)+\epsilon)} \\
& \cdot 2^{-(1-\alpha_1-\alpha_2)n(H(U,V,X_{13},X_{23})-\epsilon)} \\
& \cdot 2^{-(1-\alpha_1-\alpha_2)n(H(Y_3)-\epsilon)} \\
= & 2^{-(1-\alpha_1-\alpha_2)n(I(U,V,X_{13},X_{23};Y_3)-3\epsilon)} \\
= & 2^{-(1-\alpha_1-\alpha_2)n(I(X_{13},X_{23};Y_3)-3\epsilon)}.
\end{aligned}
$$

Therefore, $P_{E_{12}}$ can be upper-bounded as

$$
\begin{aligned}
P_{E_{12}} \leq & 2^{n(R_1+R_2)} \cdot 2^{-\alpha_1 n(I(X_{10};Y_1)-3\epsilon)-\alpha_2 n(I(X_{20};Y_2)-3\epsilon)} \cdot \\
& 2^{-(1-\alpha_1-\alpha_2)n(I(X_{13},X_{23};Y_3)-3\epsilon)}.
\end{aligned}
$$

Hence, $P_{E_{12}} \rightarrow 0$ as $n \rightarrow \infty$ if I_{10} in (4) satisfied.

C. Numerical Results

Fig. 2 shows the achievable rate region of the proposed half-duplex scheme over the Gaussian channel. The results are compared with that of the classical MAC and the full duplex scheme given in [2], [5]. In this figure, we set the transmitted signals from each user to be jointly Gaussian with total power constraint equal to 2. We also assume that the noises at the two users and the destination are i.i.d.$\sim N(0,1)$. Finally, K_{12}, and K_{21} are the inter-user channels coefficients, K_{10}, and K_{20} are the channels coefficients between each user and the destination. The results are obtained with $K_{10} = K_{20} = 1$ and different values of $K_{12} = K_{21}$. Results show that the new scheme leads to a larger rate region compared with the classical MAC. The amount of rate improvement increases with better inter-user channel quality K_{12}. Comparing to the full duplex scheme, it is expected that our scheme has a smaller rate region. However, we can see that as K_{12} increases, the two rate regions become closer till they are identical when $K_{12} \rightarrow \infty$.

V. Conclusion

In this paper, we analyzed a new cooperative half-duplex scheme where the transmission is performed over 3 time slots in each of the independent transmission blocks. We derived its achievable rate region using superposition encoding and joint typicality decoding techniques. Finally, we presented some numerical results that show the advantage of this scheme over the classical MAC.

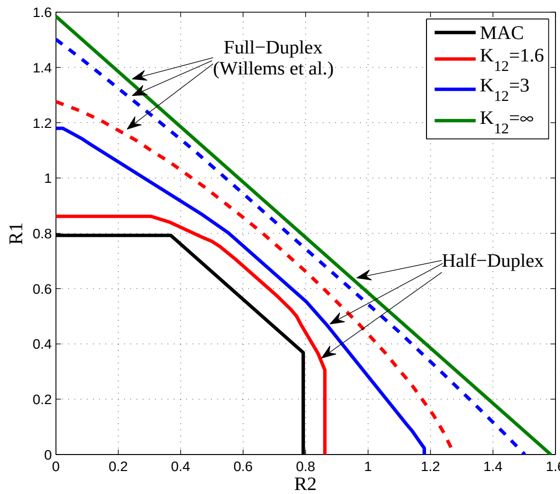

Fig. 2. Achievable rate region for half duplex cooperative scheme compared with classical MAC ($K_{10} = K_{20} = 1, K_{12} = K_{21}$).

References

[1] A. Abu Al Haija and M. Vu, “A half-duplex cooperative scheme with partial decode-forward relaying,” in *submitted to IEEE ISIT*, 2011.

[2] F. M. J. Willems, E. C. van der Meulen, and J. P. M. Schalkwijk, “Achievable rate region for the multiple-access channel with generalized feedback,” in *Proc. Annu. Allerton Conf. on Communication, Control and Computing*, 1983, pp. 284–292.

[3] T. Cover and C. Leung, “An achievable rate region for the multiple-access channel with feedback,” *IEEE Trans. Inf. Theory*, vol. IT-27, no. 3, pp. 292–298, May 1981.

[4] C. Zheng, F. Kuhlmann, and A. Buzo, “Achievability proof of some multi-user channel coding theorem using backward decoding,” *IEEE Trans. Inf. Theory*, vol. 35, no. 6, pp. 1160–1165, Nov. 1989.

[5] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity - part i&ii,” *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1948, Nov. 2003.

[6] E. C. van der Meulen, “A survey of a multi-way channels in information theory,” *IEEE Trans. Inf. Theory*, vol. 23, pp. 1–37, Jan. 1977.

[7] T. M. Cover and A. El Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. Inf. Theory*, vol. 25, pp. 572–584, Sep. 1979.

[8] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[9] J. Li, J. Ge, and X. Xiong, “Cooperative diversity based on Alamouti space-time code,” in *IEEE ICCSIT*, 2008, pp. 642–646.

[10] S. Vishwanath, S. Jafar, and S. Sandhu, “Half duplex relays: cooperative communication strategies and outer bounds,” in *IEEE International Conf. on Wireless Net., Commu. and Mobile Computing*, 2005, pp. 1455–1459.

[11] Y. Peng and D. Rajan, “Capacity bounds of half duplex Gaussian cooperative interference channel,” in *IEEE ISIT*, 2009, pp. 2081–2085.

[12] R. Wu, V. Prabhakaran, and P. Viswanath, “Interference channels with half duplex source cooperation,” in *IEEE ISIT*, 2010, pp. 375–379.

[13] A. El Gamal and S. Zahedi, “Capacity of a class of relay channels with orthogonal components,” *IEEE Trans. Inf. Theory*, vol. 51, no. 5, pp. 1815–1817, May 2005.

[14] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New York:Wiley, 2006.