Capacity- and Bayesian-Based Cognitive Sensing with Location Side Information

Peng Jia, Mai Vu, Tho Le-Ngoc, Seung-Chul Hong and Vahid Tarokh

***Abstract*—We investigate spectrum sensing by energy detection based on two different objective functions: a Bayesian sensing cost or the network weighted sum capacity. The Bayesian cost is a traditional detection measure which aims at minimizing a combination of the miss-detection and false-alarm probabilities, while the capacity objective is a communication measure which aims at maximizing the network throughput. Fading-dependent optimal sensing thresholds for each objective are derived in closed-form for different cases of location side information. To make sensing more robust to channel fading, we also propose fading-independent sub-optimal thresholds. Results show that location side information helps improve performance when using the threshold designed for that performance measure. However, the Bayesian-based threshold does not utilize the side information well in improving the network sum capacity. On the other hand, the capacity-based threshold captures the benefit of side information in both the capacity and Bayesian cost measures. Furthermore, it helps to significantly improve the network throughput. The proposed sensing schemes with location side information can also be generalized to a network with multiple cognitive users in a simple and distributed manner.**

***Index Terms*—Cognitive radio, spectrum sensing, Bayesian detection, capacity, side information.**

I. Introduction

THE RECENT demand of versatile communication services exerts large loads on the already crowded spectrum. However, the perceived spectrum shortage is mostly because of inflexible spectrum management rather than actual natural resource shortage [1]. As measurements show, some licensed bands are grossly under-utilized at various times and locations [2], resulting in spectrum holes. *Cognitive Radio*, a device with the ability to sense idle spectrum and adapt its transmission to the detected changes [3], is a promising tool for leveraging these precious spectrum holes. Since its conception, the advancement in silicon technologies makes the implementation of the cognitive radio system realistic [4]. However, much of the algorithms and prototyping is still emerging, addressing challenges from both the theoretical [5] and experimental [6] points of view.

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A fundamental issue for cognitive radios is how to keep the licensed, privileged user's communication from being impaired while still achieving its own communication successfully. To do so, the cognitive radio relies on spectrum sensing to gather information on the external environment [7], and after analyzing the gathered data, designs its respective transmission strategies. Traditional signal detection techniques have been used for spectrum sensing: matched filter, cyclostationary feature detection and energy detection [8]. Detection using matched filter can result in the best performance by maximizing the received signal-to-noise ratio, but requires a priori knowledge of the primary user's signal. Cyclostationary feature detection exploits the inherent periodical characteristics of modulated signals to compare the spectrum correlation functions, however it faces computation complexity. Energy detection [9], [10] is the simplest technique, just comparing the received energy with a predetermined threshold. Because of its simplicity and low latency, we focus on the energy detection method in this paper.

While there have been a substantial amount of research on energy detection in spectrum sensing in the past few years (see for example [11]–[13]), we approach this problem from two novel angles. First, what is the impact of different sensing criteria on the communication performance? Second, can we improve the performance of energy detection by the use of side information?

Traditional detection, often used in radar, is to detect the presence of a signal, for example, from a spacecraft. Hence the main objective is to minimize the chances of miss detections and false alarms. Miss detection occurs when there is a signal but the radar fails to detect, and false alarm occurs when no signals are present but the radar postulates one. Usually each case has an associated cost, and the detection threshold is designed to minimize a combination of these costs using a Bayesian approach. Such a design works well for what it was designed for – detecting the presence of a signal. However, when applying to a communication network, detecting the presence of the primary user's signal is only an intermediate goal. The end goal is communicating information. Hence often performance is measured not in terms of the miss-detection or false-alarm probabilities, but in terms of the transmission rate and reliability. A detection threshold designed for minimizing the miss-detection or false-alarm probabilities may not be optimal for communication. In this paper, we investigate two different objectives for designing the sensing threshold: the traditional Bayesian criterion, and a new throughput-based objective. By studying both objectives, we are able to compare and contrast the impact of different sensing threshold designs on the cognitive radio system.

Another important aspect of cognitive communications is the ability of the communicating device to process available side information, or use cognition, to improve its performance. For example, side information on the primary users' signals and code book can help increase the transmission rate region [14], [15]. Other kinds of side information are also helpful. Consider spectrum sensing in a network in which the nodes are mobile, or in which they are static but join and leave the network at random. Information on the locations of these nodes can be useful in adjusting the sensing threshold. Utilizing such location side information and understanding its impacts are the second topic studied in this paper.

The paper is organized as follows. In Section II, we introduce the spatial model for a network with a single primary and single cognitive user, and the channel model with both large- and small-scale fading. Section III formulates the two objectives for designing the sensing threshold: the network weighted sum capacity and the Bayesian cost function. Based on these two objectives, in Section IV, the optimal sensing thresholds are derived for the case of full location side information. In Section V, we extend these designs to the cases of partial location side information. Generalization to a network with multiple cognitive users is given in Section VI. Section VII presents the numerical results and our corresponding discussion. We conclude in Section VIII.

II. Network and Channel Models

A. Network model

Consider a network consisting of one cognitive user and one primary user, each having a pair of transmitter and receiver which are randomly located. The random locations can arise from mobility or random network access. Let the cognitive transmitter C_{tx} be the center of the network at the polar coordinates $(0,0)$. The cognitive receiver C_{rx} is uniformly distributed within the disc centered at the origin with radius R_c. Let the impact radius of the cognitive transmitter be R_i such that any primary receiver falling within this radius will be noticeably interfered by the cognitive transmitter. The considered primary receiver P_{rx} lies uniformly within this radius R_i. Centered at P_{rx}, the primary transmitter P_{tx} is uniformly distributed within the disc with radius R_p (see Fig. 1). The radii R_c, R_i and R_p are known network parameters.

Assume a protected region of radius ϵ centered at every receiving node. Any active transmitter cannot be inside this region to exclude the possibility that the receive signal or interference power rises to infinity.

Let S_{pt} S_{pr} and S_{cr} respectively specify the locations of P_{tx}, P_{rx} and C_{rx} within the polar coordinates. Consider S_{cr} for example, and let r denote its radius and θ denote its angle. For uniformly distributed C_{rx}, r and θ are independent and have the following probability density functions:

$$f_r(r) = \frac{2r}{R_c^2 - \epsilon^2} , \qquad \epsilon \le r \le R_c ,$$
$$f_\theta(\theta) = \frac{1}{2\pi} , \qquad 0 \le \theta \le 2\pi . \tag{1}$$

The distributions for the radius and angles in S_{pr} and S_{pt} can be similarly derived.

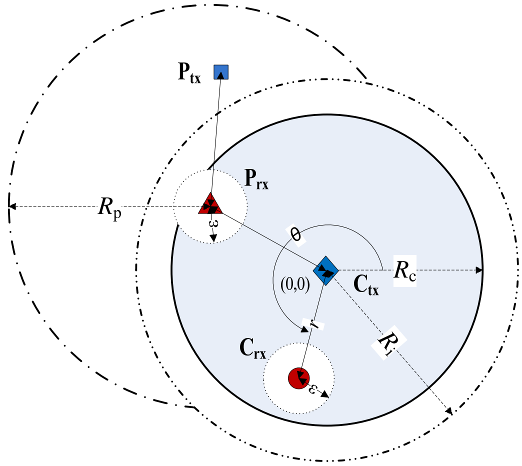

Fig. 1. Network model.

B. Channel model

Consider a wireless channel model with both large-scale path loss and small-scale fading. Free-space path loss models the averaged power changing with distance. Rayleigh fading models the small-scale variation. The channel between any transmitter-receiver (Tx-Rx) pair can then be written as

$$h = \nu \cdot \eta , \tag{2}$$

where ν is the path loss component and η is the small-scale fading component. Here we assume that $\eta \sim \mathcal{CN}(0,1)$ is a circular complex Gaussian random variable with independent, equal-variance real and imaginary parts, but our analysis also applies to other fading distributions. The pathloss component is modeled as

$$\nu = \frac{A}{d^{\alpha/2}} ,$$

where α is the pathloss exponent, d is the Tx-Rx distance and A is a constant dependent on the frequency and the transmitter/receiver antenna gain. Without loss of generality, we assume $A = 1$ in the subsequent analysis. Effectively, at a given Tx-Rx distance, the channel in (2) can also be viewed as having a circular complex Gaussian distribution with the variance dependent on the Tx-Rx distance, that is $h \sim \mathcal{CN}(0, \nu^2)$.

In the network of Figure 1, we are particularly interested in 5 channels as follows.

- h_d: the channel between P_{tx} and C_{tx} used in detection
- h_p: direct channel between P_{tx} and P_{rx}
- h_c: direct channel between C_{tx} and C_{rx}
- h_{cp}: interference channel between P_{tx} and C_{rx}
- h_{pc}: interference channel between C_{tx} and P_{rx}

All channels $h_d, h_p, h_c, h_{pc}, h_{cp}$ are independent of each other and follow the model in (2). The corresponding path loss and fading components of each channel will be denoted respectively as ν and η with the same subscript as that of the channel. We assume no channel knowledge (of either path loss or fading) at any transmitter. The implication of channel knowledge at receivers is discussed later in Section III-C.

C. Signal model

In this theoretical study, we assume that the transmit signal of either the primary or cognitive user is Gaussian with zeromean, with no particular modulations. The primary user has transmit power P_1 and the cognitive user has power P_c.

We assume that the cognitive transmitter is the device to perform sensing. When the cognitive user senses the spectrum, the cognitive transmitter is in receiving mode and tries to detect the primary's signal. If the cognitive transmitter detects no primary user's signal, it will then initiate communication with the cognitive receiver.

Let $s \in \mathcal{CN}(0, P_1)$ be the transmit signal of the primary user. When the primary user transmits, the signal received by the cognitive transmitter can be written as

$$y = h_{\mathrm{d}} s + z \tag{3}$$

where $z \sim \mathcal{CN}(0, \sigma_z^2)$ is the thermal noise. Denote the received signal part without noise as

$$x = h_{\mathrm{d}} s.$$

Note that x is the signal (without noise) received at the cognitive Tx from the primary Tx after experiencing path loss and fading.

To sense primary transmission, the cognitive transmitter needs to perform a hypothesis testing to decide between the following two hypotheses:

$$\mathcal{H}_0 : y = z \tag{4}$$

$$\mathcal{H}_1 : y = x + z\,. \tag{5}$$

For sensing, we will first make the assumption that when in detection mode, the cognitive transmitter knows perfectly the channel h_{d} from the primary transmitter. This assumption allows us to analytically derive the optimal detection threshold in closed form. Then based on the optimal threshold, we propose a sub-optimal threshold that requires at $\mathrm{C_{tx}}$ no knowledge of the small-scale fading component in h_{d} but only knowledge of the path loss component.

With the initial assumption of perfect knowledge of h_{d} at $\mathrm{C_{tx}}$, then $x \sim \mathcal{CN}(0, \sigma_x^2)$ where

$$\sigma_x^2 = |h_{\mathrm{d}}|^2 P_1. \tag{6}$$

The average of σ_x^2 over the small-scale fading is then equal to

$$\bar{\sigma}_x^2 = E_{\eta_{\mathrm{d}}}\left[|h_{\mathrm{d}}|^2 P_1\right] = \nu_{\mathrm{d}}^2 P_1. \tag{7}$$

These values will be used in designing the sensing threshold.

III. Network Capacity and Bayesian Cost Formulation

Consider spectrum sensing based on energy detection, in which the cognitive user detects the signal energy in order to decide whether to start its own transmission. In this process, a sensing threshold is set by the cognitive user to optimize a sensing objective. In this paper, we consider 2 different sensing objectives: the network weighted sum capacity, and the more traditional Bayesian cost. The capacity objective is motivated from a communication perspective, in which the transmission rate is an important performance measure. The Bayesian objective is traditionally used in radar detection to detect the presence of a signal by minimizing a combination of the false-alarm and miss-detection probabilities. In studying both of these objectives, we are able to contrast and compare the impact of objectives on the sensing performance.

A. Capacity formulation

To formulate the primary and cognitive users' rates, we first describe how the network operates. The primary user starts its communication with a certain probability λ_1. The cognitive user senses the spectrum by comparing its received signal power with a predetermined threshold. Since there is only one cognitive user, this user transmits if it perceives idle spectrum. However, the sensing decision can be wrong. When the primary user is not transmitting, wrong sensing decision gives rise to *false alarm* and the cognitive user wastes the idle bandwidth. When the primary user is transmitting, wrong decision produces *miss detection* and the cognitive user's transmission subjects both the primary user and itself to interference. Accordingly, the *ergodic capacities* of the primary user (C_{p}) and the cognitive user (C_{c}), averaged over all locations S_{cr}, S_{pr}, S_{pt} in the network and the fading in detection, can be formulated as in (8) and (9) where

- λ_1 is the transmission probability of the primary user.
- γ is the sensing threshold.
- $P(\mathcal{H}_0|\mathcal{H}_1, \gamma)$ is the miss-detection probability, when the cognitive transmitter makes a decision to transmit while the primary transmitter is also active. Denote

$$p = P(\mathcal{H}_1|\mathcal{H}_1, \gamma) = 1 - P(\mathcal{H}_0|\mathcal{H}_1, \gamma)\,. \tag{10}$$

- $P(\mathcal{H}_1|\mathcal{H}_0, \gamma)$ is the false-alarm probability, when the cognitive transmitter makes a decision not to transmit while the primary link is idle. Denote

$$q = P(\mathcal{H}_0|\mathcal{H}_0, \gamma) = 1 - P(\mathcal{H}_1|\mathcal{H}_0, \gamma)\,. \tag{11}$$

- L_{p} and L_{c} are the random received signal power at the primary and cognitive receiver as

$$L_p = |h_{\mathrm{p}}|^2 P_1\,, \quad L_c = |h_{\mathrm{c}}|^2 P_c. \tag{12}$$

- I_{pc} and I_{cp} are random interference power from the cognitive transmitter to the primary receiver, and from the primary transmitter to the cognitive receiver, respectively.

$$I_{\mathrm{pc}} = |h_{\mathrm{pc}}|^2 P_c\,, \quad I_{\mathrm{cp}} = |h_{\mathrm{cp}}|^2 P_1. \tag{13}$$

- For ergodic capacity, the innermost expectation is performed over the small-scale fading of the respective channels.

Note that the capacities as formulated in (8) and (9) are averaged over the fading component η_{d} in signal detection because the threshold depends on h_{d}. (Later, we propose a sub-optimal threshold that removes the small-scale fading dependency.) Our goal now is to maximize the network weighted throughput defined as

$$\mathcal{C} = \mu C_{\mathrm{p}} + (1-\mu) C_{\mathrm{c}} = E_{S_{\mathrm{pt}}, S_{\mathrm{pr}}, S_{\mathrm{cr}}}\, E_{\eta_{\mathrm{d}}}\left[\, ap + bq + c\,\right] \tag{14}$$

where $0 < \mu < 1$ is the weight to emphasize the importance of either the primary or cognitive link, p and q are given in (10), (11), (15), (16), and (17). Here a captures the capacity terms associated with primary user transmitting, b with primary user not transmitting, and c captures the common capacity terms.

$$C_{\mathrm{p}} = E_{S_{\mathrm{cr}},S_{\mathrm{pr}},S_{\mathrm{pt}}} E_{\eta_{\mathrm{d}}} \left\{ \lambda_1 P(\mathcal{H}_1|\mathcal{H}_1,\gamma) \cdot E\left[\log_2\left(1+\frac{L_{\mathrm{p}}}{\sigma_z^2}\right)\right] + \lambda_1 P(\mathcal{H}_0|\mathcal{H}_1,\gamma) \cdot E\left[\log_2\left(1+\frac{L_{\mathrm{p}}}{\sigma_z^2+I_{\mathrm{pc}}}\right)\right]\right\} \quad (8)$$

$$C_{\mathrm{c}} = E_{S_{\mathrm{cr}},S_{\mathrm{pr}},S_{\mathrm{pt}}} E_{\eta_{\mathrm{d}}} \left\{ (1-\lambda_1) P(\mathcal{H}_0|\mathcal{H}_0,\gamma) \cdot E\left[\log_2\left(1+\frac{L_{\mathrm{c}}}{\sigma_z^2}\right)\right] + \lambda_1 P(\mathcal{H}_0|\mathcal{H}_1,\gamma) \cdot E\left[\log_2\left(1+\frac{L_{\mathrm{c}}}{\sigma_z^2+I_{\mathrm{cp}}}\right)\right]\right\} \quad (9)$$

$$a = \mu\,\lambda_1 E\left[\log_2\left(1+\frac{L_{\mathrm{p}}}{\sigma_z^2}\right)\right] - \mu\,\lambda_1 E\left[\log_2\left(1+\frac{L_{\mathrm{p}}}{\sigma_z^2+I_{\mathrm{pc}}}\right)\right] - (1-\mu)\lambda_1 E\left[\log_2\left(1+\frac{L_{\mathrm{c}}}{\sigma_z^2+I_{\mathrm{cp}}}\right)\right] \quad (15)$$

$$b = (1-\mu)(1-\lambda_1) E\left[\log_2\left(1+\frac{L_{\mathrm{c}}}{\sigma_z^2}\right)\right] \quad (16)$$

$$c = \mu\,\lambda_1 E\left[\log_2\left(1+\frac{L_{\mathrm{p}}}{\sigma_z^2+I_{\mathrm{pc}}}\right)\right] + (1-\mu)\lambda_1 E\left[\log_2\left(1+\frac{L_{\mathrm{c}}}{\sigma_z^2+I_{\mathrm{cp}}}\right)\right] \quad (17)$$

B. Bayesian cost formulation

Another objective which is common in spectrum sensing involves minimizing the *false-alarm* and *miss-detection* probabilities. In this section, we consider a Bayesian cost function as a linear combination of both probabilities as

$$\Re = R_{\mathrm{m}} P(\mathcal{H}_0|\mathcal{H}_1,\gamma)\lambda_1 + R_{\mathrm{f}} P(\mathcal{H}_1|\mathcal{H}_0,\gamma)(1-\lambda_1)\,, \quad (18)$$

where R_{m} is the cost associated with miss detection and R_{f} is the cost associated with false alarm. These two costs are designed to take into consideration the effects on interaction in the network as follows.

Since when miss detection occurs, both links interfere with each other, R_{m} can be set as

$$R_{\mathrm{m}} = \beta(\bar{I}_{\mathrm{pc}} + \bar{I}_{\mathrm{cp}})\,, \quad (19)$$

where $\beta \geqslant 1$ is a penalty parameter to emphasize the cost of miss detection, hence placing a higher priority on the primary link, and $\bar{I}_{\mathrm{pc}}$ and $\bar{I}_{\mathrm{cp}}$ are the averages over the small-scale fading of the interference powers in (13).

When a false alarm occurs, the cognitive user misses its chance to achieve a higher rate, thus R_{f} can be set as the average power $\bar{L}_c$ received by the cognitive receiver

$$R_{\mathrm{f}} = \bar{L}_c\,. \quad (20)$$

Here $\bar{L}_c$ is average of L_c in (12) over the small-scale fading (that is, $\bar{L}_c = E_{\eta_{\mathrm{c}}}[L_c]$).

Taking into account the random locations of the transmitters and receivers and the channel fading in detection, the cost in (18) can then be written as

$$\Re = E_{S_{\mathrm{cr}},S_{\mathrm{pr}},S_{\mathrm{pt}}} E_{\eta_{\mathrm{d}}} \Big[\beta(\bar{I}_{\mathrm{pc}} + \bar{I}_{\mathrm{cp}})\lambda_1 P(\mathcal{H}_0|\mathcal{H}_1,\gamma) + \bar{L}_c(1-\lambda_1)P(\mathcal{H}_1|\mathcal{H}_0,\gamma)\Big]\,. \quad (21)$$

In this formulation, the goal is to minimize this Bayesian cost function.

C. Channel knowledge requirements

We have set up the two different sensing objectives. The capacity objective (14) is a non-linear function of the received signal and interference powers, whereas the Bayesian cost objective (21) is a linear combination of these power terms. Each objective will be used to design the correspondingly optimal sensing threshold.

1) Channel knowledge in each formulation: In both formulations, we assume no channel state information at the transmitters. The Bayesian formulation also requires no knowledge of instantaneous channel realizations at the receiver side, but only knowledge of the channel distribution. The costs in (19) and (20) are received or interference power averaged over fading. For the capacity formulation, even though all capacity terms in (8) and (9) are ergodic which is averaged over fading, each receiver need to know the instantaneous channel from its corresponding transmitter for Shannon capacity formula to apply. Specifically, the primary receiver need to know h_{p} and the cognitive receiver to know h_{c}. However, since interference is treated as noise, each receiver is not required to know the interference channel – the receiver only estimates the composite noise and interference power.

In performing detection and hence computing the miss-detection or false-alarm probabilities (in (8), (9) and (18)), the received signal power at the cognitive transmitter is compared with the threshold, but obtaining this received power does not require knowing the channel realizations. (This would only be required if the cognitive user wanted to estimate the primary transmit signal, but here it merely wants to detect the presence of this primary signal.) For threshold computation, as mentioned earlier, we first assume instantaneous knowledge of channel h_{d}, but then remove the requirement of instantaneous knowledge of the small-scale fading component as discussed later in Section IV-C. Thus performing detection will require only knowledge of the channel distribution.

In short, the Bayesian formulation only relies on knowing the fading distributions at the receivers. But the capacity formulation requires each receiver to know the instantaneous fading realizations from its own transmitter. This instantaneous channel knowledge, however, is often already in-place for received signal estimation.

2) Obtaining channel distribution for detection: Channel distributions can be obtained based on channel models or on long-term measurement of the channels, which can be carried out in advance for a specific environment. In case the cognitive user wishes to concurrently measure the fading distribution between the primary transmitter and the cognitive transmitter, several methods may be employed. One can be that the cognitive user measures the fading distribution of the channel between its own transmitter and receiver, using its own training sequences, and assume the same fading distribution for the channel from the primary user, who transmits in the same environment. Another can be that the cognitive user "piggybacks" on the primary user's training sequences when the primary users is detected to be active. For primary systems built according to a certain standard, this method is plausible since primary training sequences are known a priori. Furthermore, the cognitive user only uses these sequences to measure the channel distribution, not the instantaneous channel.

In this paper, we assume Rayleigh fading (2), but the analysis and subsequent threshold designs are also applicable to other fading distributions.

IV. Thresholds with Full Location Side Information

From the point of view of the cognitive transmitter, the locations of the other three radios in the network are side information that can be used to help design the sensing threshold. However, these locations may or may not be known to the cognitive transmitter. In this section, we consider the case of full knowledge of location information, i.e., C_{tx} knows the locations of its own receiver and of both the primary transmitter and receiver. Then in the Section V, we will explore the partial information cases.

A. Optimal threshold for the capacity sensing objective

With full location side information, maximizing the objective (14) is equivalent to maximizing $ap + bq + c$ for each realization of $\{S_{\rm pt}, S_{\rm pr}, S_{\rm cr}\}$. The signal power received at the cognitive Tx from the primary Tx follows the two degree chi-square distribution in both primary transmitting and non-transmitting situations with different variances. Thus, for each set of $\{S_{\rm pt}, S_{\rm pr}, S_{\rm cr}\}$, we can express the objective function as

$$\begin{aligned} & ap + bq + c \\ &= a \int_{\gamma}^{+\infty} \frac{1}{\sigma_z^2 + \sigma_x^2} \exp\left(-\frac{\xi}{\sigma_z^2 + \sigma_x^2}\right) d\xi \\ &\quad + b \int_0^{\gamma} \frac{1}{\sigma_z^2} \exp\left(-\frac{\xi}{\sigma_z^2}\right) d\xi + c \\ &= a \exp\left(-\frac{\gamma}{\sigma_z^2 + \sigma_x^2}\right) - b \exp\left(-\frac{\gamma}{\sigma_x^2}\right) + b + c \\ &= f(\gamma). \end{aligned} \tag{22}$$

Our goal is to find $\gamma \geq 0$ to maximize $f(\gamma)$. From (15) and (16), we observe that the coefficient b is always positive, but a can be non-positive in some locations under certain primary transmission probabilities.

When $a \leqslant 0$, to maximize (22), the threshold should be set to infinity, implying the cognitive radio always transmits.

When $a > 0$, take the first derivative of (22) as

$$f'(\gamma) = -\frac{a}{\sigma_z^2 + \sigma_x^2} \exp\left(-\frac{\gamma}{\sigma_z^2 + \sigma_x^2}\right) + \frac{b}{\sigma_z^2} \exp\left(-\frac{\gamma}{\sigma_z^2}\right).$$

Equating this derivative to zero, we obtain an optimal point as

$$\gamma = \left(\sigma_z^2 + \sigma_x^2\right) \frac{\sigma_z^2}{\sigma_x^2} \ln\left(\frac{\sigma_z^2 + \sigma_x^2}{\sigma_z^2} \frac{b}{a}\right). \tag{23}$$

To ensure that this point is the maximum, obtain the second derivative of (22) as

$$f''(\gamma) = \frac{a}{(\sigma_z^2 + \sigma_x^2)^2} \exp\left(-\frac{\gamma}{\sigma_z^2 + \sigma_x^2}\right) - \frac{b}{\sigma_z^4} \exp\left(-\frac{\gamma}{\sigma_z^2}\right).$$

To be a maximum point, the second derivation should be negative at this point, thus,

$$\gamma < \left(\sigma_z^2 + \sigma_x^2\right) \frac{\sigma_z^2}{\sigma_x^2} \ln\left[\left(\frac{\sigma_z^2 + \sigma_x^2}{\sigma_z^2}\right)^2 \frac{b}{a}\right]. \tag{24}$$

The optimal value (23) always satisfies condition (24), hence $f(\gamma)$ is maximized at this value.

Thus the optimal sensing threshold can be obtained as

$$\gamma_1^{\rm c} = \begin{cases} \infty, & \text{if } a \leqslant 0, \\ \left(\sigma_z^2 + \sigma_x^2\right) \frac{\sigma_z^2}{\sigma_x^2} \ln\left(\frac{\sigma_z^2 + \sigma_x^2}{\sigma_z^2} \frac{b}{a}\right) & \text{if } a > 0. \end{cases} \tag{25}$$

For some set of locations, $\gamma_1^{\rm c}$ can be negative, implying that C_{tx} always stays silent. This situation can happen, for example, when the primary users activity factor is high (λ_1 is near 1) and the cognitive transmitter is close to the primary receiver, or the cognitive receiver is close to the primary transmitter (so that the interference $I_{\rm pc}$ or $I_{\rm cp}$ is high). In such a case, the value of a in (15) will be large and the value of b in (16) will be small, causing the threshold in (25) to be negative.

The capacity-optimal threshold can thus be written as

$$\gamma^\star = \max\left(0, \gamma_1^{\rm c}\right). \tag{26}$$

The network weighted sum capacity can be computed using this optimal threshold for each realization of $\{S_{\rm pr}, S_{\rm cr}, S_{\rm pt}\}$ and $\eta_{\rm d}$, then averaged over all location and $\eta_{\rm d}$ realizations.

B. Optimal threshold for the Bayesian sensing objective

For the Bayesian sensing objective, minimizing the cost in (21) is equivalent to minimizing this Bayesian cost for each specific set of locations $S_{\rm pr}$, $S_{\rm cr}$ and $S_{\rm pt}$ as

$$\begin{aligned} \Re_1 = &\beta\, (\bar{I}_{\rm pc} + \bar{I}_{\rm cp})\lambda_1\, P(\mathcal{H}_0|\mathcal{H}_1, \gamma) \\ &+ \bar{L}_c(1 - \lambda_1)\, P(\mathcal{H}_1|\mathcal{H}_0, \gamma). \end{aligned} \tag{27}$$

Taking into account the complex Gaussian distribution of the received signal y in both hypotheses (4) and (5), the optimal decision rule in the Bayesian criterion is

$$\begin{aligned} &\frac{\beta\, (\bar{I}_{\rm pc} + \bar{I}_{\rm cp})\lambda_1\, P(|y| \mid \mathcal{H}_1)}{\bar{L}_{\rm c}(1 - \lambda_1)\, P(|y| \mid \mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} 1 \\ \Longleftrightarrow\; &\frac{\frac{2|y|}{\sigma_x^2 + \sigma_z^2}\, e^{-\frac{|y|^2}{\sigma_x^2 + \sigma_z^2}}}{\frac{2|y|}{\sigma_z^2}\, e^{-\frac{|y|^2}{\sigma_z^2}}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \frac{1 - \lambda_1}{\lambda_1} \frac{\bar{L}_{\rm c}}{\beta\big(\bar{I}_{\rm pc} + \bar{I}_{\rm cp}\big)}. \end{aligned} \tag{28}$$

Thus the optimal threshold can be computed as

$$\begin{aligned} \gamma_1^{\rm b} = \frac{\sigma_z^2}{\sigma_x^2}(\sigma_x^2 + \sigma_z^2) \Bigg[&\ln\left(1 + \frac{\sigma_x^2}{\sigma_z^2}\right) + \ln\left(\frac{1 - \lambda_1}{\lambda_1}\right) \\ &+ \ln\left(\frac{\bar{L}_{\rm c}}{\beta\left(\bar{I}_{\rm pc} + \bar{I}_{\rm cp}\right)}\right)\Bigg]. \end{aligned} \tag{29}$$

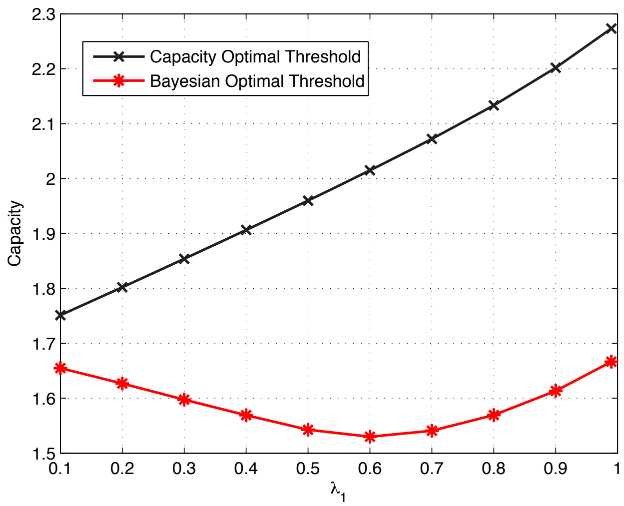

Fig. 2. Network throughput for two different sensing thresholds.

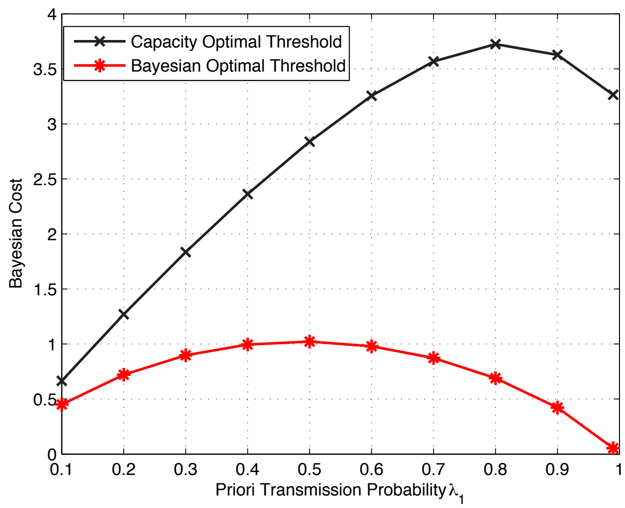

Fig. 3. Bayesian cost for two different sensing thresholds.

For the same reason as in the capacity-optimal threshold, the Bayesian-optimal threshold can be written as $\gamma^\star = \max\left(0, \gamma_1^{\mathrm{b}}\right)$. For either threshold, the detection decision follows as $|y|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma$.

C. Sub-optimal thresholds independent of channel fading

The optimal thresholds in (25) and (29) are function of σ_x^2, which depends on the realization of channel h_d as in (6). Since obtaining the instantaneous realization of h_d can be a cumbersome task for the cognitive transmitter, here we propose sub-optimal thresholds that only requires knowledge of the channel distribution (that is, only knowledge of ν_d but not η_d). Specifically, we replace σ_x^2 in each threshold expression with $\bar{\sigma}_x^2$ averaged over fading as in (7).

Then the sub-optimal threshold can be obtained as $\tilde{\gamma}^\star = \max(0, \tilde{\gamma}_1)$, where

- for the capacity objective,

$$\tilde{\gamma}_1 = \begin{cases} \infty, & \text{if } a \leqslant 0, \\ \left(\sigma_z^2 + \bar{\sigma}_x^2\right) \dfrac{\sigma_z^2}{\bar{\sigma}_x^2} \ln\left(\dfrac{\sigma_z^2 + \bar{\sigma}_x^2}{\sigma_z^2} \dfrac{b}{a}\right) & \text{if } a > 0. \end{cases} \tag{30}$$

- for the Bayesian objective,

$$\tilde{\gamma}_1 = \frac{\sigma_z^2}{\bar{\sigma}_x^2}(\bar{\sigma}_x^2 + \sigma_z^2)\left[\ln\left(1 + \frac{\bar{\sigma}_x^2}{\sigma_z^2}\right) + \ln\left(\frac{1-\lambda_1}{\lambda_1}\right) + \ln\left(\frac{\bar{L}_\mathrm{c}}{\beta\left(\bar{I}_\mathrm{pc} + \bar{I}_\mathrm{cp}\right)}\right)\right]. \tag{31}$$

When computing the capacity in (14) or Bayesian cost in (21) using sub-optimal threshold $\tilde{\gamma}^\star$, we no longer the need to take the average over η_d.

Figure 2 shows an example of the network throughput in (14) (with $\mu = 0.5$) obtained using both thresholds (30) and (31), while Figure 3 shows the corresponding Bayesian costs in (21) (with $\beta = 1$). (In Section VII, we provide more results on the effect of varying μ.) These plots show that the capacity-based threshold improves the network throughput significantly compared to the Bayesian-based threshold, while the Bayesian-based threshold has an advantage in reducing the miss-detection and false-alarm probabilities. A consequent question becomes: is it better to design the sensing threshold to minimize the miss-detection and false-alarm probabilities or to directly optimize the network throughput? The probabilities of *miss detection* and *false alarm* are just intermediate performance indicators, while the ultimate goal for a network should be to operate at its maximum capacity. These results reveal that Bayesian-based detection does not always imply optimal network capacity.

V. Thresholds with Partial Location Side Information

As shown in Section IV, the locations $S_\mathrm{pr}, S_\mathrm{cr}, S_\mathrm{pt}$ can be used as side information by the cognitive transmitter to adjust its sensing threshold accordingly. The intuition is that better knowledge of location information can help improve performance. To investigate this point, in this section, we consider the following three cases of partial location side information and one case of no information:

(A) C_{tx} knows S_pr, S_pt,
(B) C_{tx} knows S_cr, S_pt,
(C) C_{tx} only knows S_pt,
(D) C_{tx} has no location information.

Except for case (D), these cases of partial location information are different from the cases that we studied previously in [16] and [17]. In all cases, we assume that σ_x^2 and σ_z^2 are known, as these values can be estimated from stored sampled received signals. Under this assumption, based on the channel model, the cognitive transmitter is able to use σ_x^2 to deduce its distance to the primary transmitter, but not the angle coordination of the primary transmitter and hence, cannot know the primary transmitter's location precisely. For this reason, the chosen partial location information cases (A), (B) and (C) are designed such that the primary transmitter's location is always known, thus being consistent with knowing σ_x^2 and allowing the full use available information. Knowing the location of the primary transmitter is also feasible in practice since the transmitter is an active device. A challenge, however, in assuming the knowledge of S_{pt} in our network model of Figure 1 is that instead of using the marginal distribution of S_{pr} as in (1), we have to use the conditional distribution of S_{pr} given S_{pt}, which is difficult to obtain analytically. This conditional distribution is not uniform in the disc centered at C_{tx}. Later in simulations, we resolve to numerical methods to compute this conditional distribution. The last case (D) is chosen for a comparison basis.

A. When S_{pr} and S_{pt} are known

Since C_{tx} has no knowledge of S_{cr}, it should design the threshold based on the objective function averaged over S_{cr} for each pair of $\{S_{\text{pr}}, S_{\text{pt}}\}$.

For the capacity objective, maximizing (14) is equivalent to maximizing

$$\mathcal{C}_2 = E_{S_{\text{cr}}}[a] \int_{\gamma}^{+\infty} \frac{1}{2(\sigma_z^2+\sigma_x^2)}\, e^{-\frac{y}{2(\sigma_z^2+\sigma_x^2)}}\, dy + E_{S_{\text{cr}}}[b] \int_0^{\gamma} \frac{1}{2\sigma_z^2}\, e^{-\frac{y}{2\sigma_z^2}}\, dy + E_{S_{\text{cr}}}[c]\,.$$

Then following similar optimal derivation as in Section IV-A and sub-optimal approximation as in Section IV-C, we obtain the capacity-based threshold for each pair of $\{S_{\text{pr}}, S_{\text{pt}}\}$ as $\tilde{\gamma}^\star = \max(0, \tilde{\gamma}_2^{\text{c}})$, where

$$\tilde{\gamma}_2^{\text{c}} = \begin{cases} \infty, & \text{if } E_{S_{\text{cr}}}[a] \leqslant 0, \\ (\sigma_z^2+\bar{\sigma}_x^2)\dfrac{\sigma_z^2}{\bar{\sigma}_x^2}\ln\left(\dfrac{\sigma_z^2+\bar{\sigma}_x^2}{\sigma_z^2}\,\dfrac{E_{S_{\text{cr}}}[b]}{E_{S_{\text{cr}}}[a]}\right) & \text{if } E_{S_{\text{cr}}}[a] > 0. \end{cases} \tag{32}$$

The difference between this case and the full location information case is that in the threshold, the capacity variables a and b are averaged over the unknown location, and whether the threshold is set to infinity depends on the average of a.

For the Bayesian cost objective, minimizing (18) is equivalent to minimizing

$$\Re_2 = \beta\left(\bar{I}_{\text{pc}} + E_{S_{\text{cr}}}[\bar{I}_{\text{cp}}]\right)\lambda_1\, P(\mathcal{H}_0|\mathcal{H}_1,\gamma) + E_{S_{\text{cr}}}[\bar{L}_{\text{c}}](1-\lambda_1)\, P(\mathcal{H}_1|\mathcal{H}_0,\gamma)\,.$$

Similar to the analysis in Sections IV-B and IV-C, the Bayesian-based threshold can be derived as $\tilde{\gamma}^\star = \max(0, \tilde{\gamma}_2^{\text{b}})$, where

$$\tilde{\gamma}_2^{\text{b}} = \frac{\sigma_z^2}{\bar{\sigma}_x^2}\left(\bar{\sigma}_x^2+\sigma_z^2\right)\left[\ln\left(1+\frac{\bar{\sigma}_x^2}{\sigma_z^2}\right) + \ln\left(\frac{1-\lambda_1}{\lambda_1}\right) + \ln\left(\frac{E_{S_{\text{cr}}}[\bar{L}_{\text{c}}]}{\beta(\bar{I}_{\text{pc}} + E_{S_{\text{cr}}}[\bar{I}_{\text{cp}}])}\right)\right]. \tag{33}$$

The difference between this threshold and the one with full location information is that the power variables $\bar{L}_{\text{c}}$, $\bar{I}_{\text{pc}}$ and $\bar{I}_{\text{cp}}$ are averaged over the unknown location.

Since the threshold γ is now derived for each pair $\{S_{\text{pr}}, S_{\text{pt}}\}$, the detection probabilities $P(\mathcal{H}_0|\mathcal{H}_0,\gamma)$ and $P(\mathcal{H}_0|\mathcal{H}_1,\gamma)$ are also for a specific $\{S_{\text{pr}}, S_{\text{pt}}\}$ pair. Thus to compute the capacities of the primary and cognitive users, for each threshold γ, we need to average the affected $\log$ terms over the unknown location S_{cr}, together with averaging over the channel fading. In this case, S_{cr} only affects the cognitive user's capacity through the received power L_{c}, hence the cognitive user capacity is computed as

$$C_{\text{c}} = E_{S_{\text{pr}},S_{\text{pt}}}\left[\lambda_1 P(\mathcal{H}_0|\mathcal{H}_0,\gamma)\cdot E_{S_{\text{cr}}}\left\{E\left[\log_2\left(1+\frac{L_{\text{c}}}{\sigma_z^2}\right)\right]\right\} + \lambda_1 P(\mathcal{H}_0|\mathcal{H}_1,\gamma)\cdot E_{S_{\text{cr}}}\left\{E\left[\log_2\left(1+\frac{L_{\text{c}}}{\sigma_z^2+I_{\text{cp}}}\right)\right]\right\}\right]. \tag{34}$$

Computing the primary user's capacity is unchanged from (8), but without expectation over η_{d}.

B. When S_{cr} and S_{pt} are known

Similar to case (A), since C_{tx} does not know S_{pr}, the affected terms in each objective function should be averaged over all S_{pr} for each given pair $\{S_{\text{cr}}, S_{\text{pt}}\}$. The threshold for each pair $\{S_{\text{cr}}, S_{\text{pt}}\}$ can be shown to be $\tilde{\gamma}^\star = \max(0, \tilde{\gamma}_3)$ where

- for the capacity objective, see (35)
- for the Bayesian cost objective,

$$\tilde{\gamma}_3 = \frac{\sigma_z^2}{\bar{\sigma}_x^2}(\bar{\sigma}_x^2+\sigma_z^2)\left[\ln\left(1+\frac{\bar{\sigma}_x^2}{\sigma_z^2}\right) + \ln\left(\frac{1-\lambda_1}{\lambda_1}\right) + \ln\left(\frac{\bar{L}_{\text{c}}}{\beta\left(E_{S_{\text{pr}}}[\bar{I}_{\text{pc}}] + \bar{I}_{\text{cp}}\right)}\right)\right]. \tag{36}$$

In this case, S_{pr} only affects the primary link capacity through the received power L_{p}, so the primary user's capacity is computed as

$$C_{\text{p}} = E_{S_{\text{cr}},S_{\text{pt}}}\left[\lambda_1 P(\mathcal{H}_1|\mathcal{H}_1,\gamma)\cdot E_{S_{\text{pr}}}\left\{E\left[\log_2\left(1+\frac{L_{\text{p}}}{\sigma_z^2}\right)\right]\right\} + \lambda_1 P(\mathcal{H}_0|\mathcal{H}_1,\gamma)\cdot E_{S_{\text{pr}}}\left\{E\left[\log_2\left(1+\frac{L_{\text{p}}}{\sigma_z^2+I_{\text{pc}}}\right)\right]\right\}\right], \tag{37}$$

while the cognitive user's capacity is computed as in (9) (without expecting over η_{d}).

C. When only S_{pt} is known

As C_{tx} only has information of S_{pt}, the affected terms in the objective functions are averaged over both S_{cr} and S_{pr}. The threshold for each S_{pt} can be shown to be $\tilde{\gamma}^\star = \max(0, \tilde{\gamma}_4)$ where

- for the capacity objective, see (38)
- for the Bayesian cost objective,

$$\tilde{\gamma}_4 = \frac{\sigma_z^2}{\bar{\sigma}_x^2}(\bar{\sigma}_x^2+\sigma_z^2)\left[\ln\left(1+\frac{\bar{\sigma}_x^2}{\sigma_z^2}\right) + \ln\left(\frac{1-\lambda_1}{\lambda_1}\right) + \ln\left(\frac{E_{S_{\text{cr}}}[\bar{L}_{\text{c}}]}{\beta\left(E_{S_{\text{pr}}}[\bar{I}_{\text{pc}}] + E_{S_{\text{cr}}}[\bar{I}_{\text{cp}}]\right)}\right)\right].$$

In this case, since S_{cr} and S_{pr} are not known, $P(\mathcal{H}_0|\mathcal{H}_0,\gamma)$ and $P(\mathcal{H}_0|\mathcal{H}_1,\gamma)$ are specific for each S_{pt}. The primary and cognitive capacities can be computed as in (37) and (34), respectively.

$$\tilde{\gamma}_3 = \begin{cases} \infty, & \text{if } E_{S_{\text{pr}}}[a] \leqslant 0 \ , \\ (\sigma_z^2 + \bar{\sigma}_x^2)\dfrac{\sigma_z^2}{\bar{\sigma}_x^2} \ \ln\left(\dfrac{\sigma_z^2 + \bar{\sigma}_x^2}{\sigma_z^2} \dfrac{E_{S_{\text{pr}}}[b]}{E_{S_{\text{pr}}}[a]}\right) & \text{if } E_{S_{\text{pr}}}[a] > 0 \ ; \end{cases} \tag{35}$$

$$\tilde{\gamma}_4 = \begin{cases} \infty, & \text{if } E_{S_{\text{cr}},S_{\text{pr}}}[a] \leqslant 0 \ , \\ (\sigma_z^2 + \bar{\sigma}_x^2)\dfrac{\sigma_z^2}{\bar{\sigma}_x^2} \ \ln\left(\dfrac{\sigma_z^2 + \bar{\sigma}_x^2}{\sigma_z^2} \dfrac{E_{S_{\text{cr}},S_{\text{pr}}}[b]}{E_{S_{\text{cr}},S_{\text{pr}}}[a]}\right) & \text{if } E_{S_{\text{cr}},S_{\text{pr}}}[a] > 0 \ ; \end{cases} \tag{38}$$

D. When no location information is available

In this case, both objective functions are averaged over all the locations. There is only a single threshold for all sets of locations. The capacity-based threshold is obtained from (30) by averaging a and b over all locations. Similarly, the Bayesian-based threshold is obtained from (31) by averaging each of the variables $\bar{L}_{\text{c}}$, $\bar{I}_{\text{pc}}$ and $\bar{I}_{\text{cp}}$ over all locations. The capacity of each user can be computed as in (8) and (9) but without execution over η_{d}, using a single threshold for all locations.

VI. Generalization to a Network with Multiple Cognitive Users

For a network with multiple cognitive users, there are two separate issues: how each user senses the spectrum, and how all the users then use the spectrum sensing results to communicate. The first issue relates to the sensing technique performed by each user, whereas the second relates to the network protocol among the cognitive users. In this paper, we provide an answer to the first issue by showing that the proposed sensing designs can be extended to multiple cognitive users in a distributed manner. The second topic is outside the scope of this paper, and to this extent, we only provide some discussion and pointers to other references.

A. Sensing technique

Consider a cognitive network with n cognitive users who wish to sense the spectrum. The proposed sensing designs can be extended to this case in a distributed manner. Each cognitive user only needs to use local information on the locations of its own receiver and the primary transmitter and receiver. Each cognitive radio can sense the spectrum independently of others; in this way, they do not need to synchronize.

The interference from other cognitive users is averaged over fading and all locations and is treated as additional background noise. To account for the fact that not all cognitive users may be active, we introduce an active factor $\delta < 1$, such that the number of cognitive user concurrently transmitting is δn. The specific δ value can be obtained through analytical modeling or simulation. Assume that all cognitive users follow the same spatial distribution as introduced in Section II. Specifically, from the perspective of each cognitive transmitter (independent of other cognitive users), the primary receiver is uniformly distributed within a disc centered at it, and its cognitive receiver is also uniformly distributed within a disc of different radius. (The analysis also applies if we instead choose the primary receiver to be the origin, so that all cognitive transmitters are uniformly distributed around the primary receiver, possibly within different radii, and each cognitive receiver is uniformly distributed around its respective cognitive transmitter.) A cognitive transmitter can then compute the average interference power from other active cognitive users and treat that as additional noise in its threshold calculation. The effective noise powers at the primary receiver, cognitive receiver and cognitive transmitter respectively become

$$\begin{aligned} \sigma_{\text{pr}}^2 &= \sigma_z^2 + \delta\, n\, \hat{I}_{\text{pc}} \\ \sigma_{\text{cr}}^2 &= \sigma_z^2 + \delta(n-1)\hat{I}_{\text{cr}} \\ \sigma_{\text{ct}}^2 &= \sigma_z^2 + \delta(n-1)\hat{I}_{\text{ct}} \end{aligned} \tag{39}$$

where $\hat{I}_{\text{pc}}$, $\hat{I}_{\text{cr}}$ and $\hat{I}_{\text{ct}}$ are respectively the *average interference* (averaged over fading and all locations) from each cognitive transmitter to the primary receiver, and from each interfering cognitive transmitter to a specific cognitive receiver and transmitter.

This way of extension allows each cognitive user to use the proposed threshold designs directly and independently of other cognitive users. Since the interference is averaged over fading and over all locations, each cognitive user only needs to know the total number of cognitive radios and their activity factor in order to compute the effective noise power (39). This effective noise is then used for designing the sensing threshold locally at each cognitive radio, using the proposed designs.

As an example, we establish the capacity sensing objective for each user. The Bayesian cost objective can be derived similarly. For the i-th cognitive user, the capacity sensing objective now becomes (40). Here L_{c}^i is the received signal power at cognitive receiver i from its own transmitter, and I_{pc}^i (I_{cp}^i) is the interference power at the primary receiver (cognitive receiver i) from the cognitive transmitter i (primary transmitter). Note that because of the averaged interferences in the effective noises, (40) as defined is not the same as the weighted sum capacity averaged over the locations of all cognitive users, which would require the instantaneous interferences from all other users and then averaging the resulting capacity over those user locations. However, we choose this objective function because it allows a distributed application of the proposed sensing designs, without requiring the knowledge of the locations of other cognitive users. The sensing threshold for cognitive user i can then be designed based on the solutions for the single primary and single cognitive user case in Sections IV and V.

$$E_{S_{\rm cr}^i,S_{\rm pr},S_{\rm pt}}\, E_{\eta_{\rm d}^i}[a_i p_i + b_i q_i + c_i], \tag{40}$$

where

$$\begin{aligned}
a_i =&\ \mu\,\lambda_1\left(E\left[\log_2\left(1+\frac{L_{\rm p}}{\sigma_{\rm pr}^2}\right)\right] - E\left[\log_2\left(1+\frac{L_{\rm p}}{\sigma_{\rm pr}^2+I_{\rm pc}^i}\right)\right]\right) - (1-\mu)\lambda_1 E\left[\log_2\left(1+\frac{L_{\rm c}^i}{\sigma_{\rm cr}^2+I_{\rm cp}^i}\right)\right],\\
b_i =&\ (1-\mu)(1-\lambda_1)E\left[\log_2\left(1+\frac{L_{\rm c}^i}{\sigma_{\rm cr}^2}\right)\right],\\
c_i =&\ \mu\,\lambda_1 E\left[\log_2\left(1+\frac{L_{\rm p}}{\sigma_{\rm pr}^2+I_{\rm pc}^i}\right)\right] + (1-\mu)\lambda_1 E\left[\log_2\left(1+\frac{L_{\rm c}^i}{\sigma_{\rm cr}^2+I_{\rm cp}^i}\right)\right],
\end{aligned}$$

and

$$p_i = P(\mathcal{H}_1|\mathcal{H}_1,\gamma_i)\ , \quad q_i = P(\mathcal{H}_0|\mathcal{H}_0,\gamma_i) \tag{41}$$

B. Cognitive network operating protocol

Given the sensing result at each cognitive user, how these users then use this result to communicate, and their subsequent performance, depend on the network protocol among the cognitive users. This topic is outside the scope of our paper, here we only discuss briefly.

If each cognitive user transmits independently of other cognitive users, according to its own sensing decision, then as the number of cognitive users increases, how the network throughput behaves becomes a question of scaling law. This is an entirely different focus which has been studied extensively for homogeneous networks (see [18], [19]), and more recently for cognitive networks [20]. For example, the throughput may scale as squared root of the number of users. Studying of scaling law requires asymptotic analysis that depends not only on the spatial distribution of the cognitive users but also on how they process and forward signals.

On the other hand, if the cognitive users have a specific network protocol that dictates who (or how many) to transmit if the spectrum is sensed vacant, or how to collaborate among themselves, then the performance will depend on this protocol. There have been sizable research efforts on collaborative sensing and access policy for secondary spectrum access (see for example [12], [21]).

As an example, suppose each cognitive user independently transmits with a certain activity factor based on its own sensing decision, then the resulted network throughput, averaged over all users locations, can be computed as in (42). In this expression, η denotes all channel fading, $\mathbf{S}$ is the set of locations of the primary Tx, Rx and all cognitive Tx's and Rx's, $J \subset \{1,\ldots,n\}$ is the subset of active cognitive users, p_i and q_i respectively are as defined in (41) (which are one minus the probabilities of miss detection and false alarm of cognitive user i respectively), and $I_{\rm cc}^{jl}$ is the interference from cognitive transmitter l to cognitive receiver j. The first summation term in (42) denotes the average capacity of the primary user, taken into account all combinations of miss detection by the cognitive users. The second summation is the average capacity obtained by all cognitive users who miss-detected, and the third term is by cognitive users without miss detection nor false alarm. Later in simulation, we plot this throughput for a network with 2 cognitive users, showing that the proposed sensing schemes when applied to a multiple cognitive user network also capture the benefit of local location information.

VII. Numerical Results

A. Simulation methods

1) Single cognitive single primary network: For simulations, we use the model in Figure 1 and set the network radii $R_{\rm c} = R_{\rm p} = R_{\rm i} = 10$, the protection region $\epsilon = 1$ and the path loss parameter $\alpha = 2.1$. The primary and cognitive transmit power and the thermal noise are set such that at the edge of a disc, the SNR is 0dB. For the case of full knowledge of location information, we first generate 3000 sets of locations $S_{\rm pt}, S_{\rm pr}, S_{\rm cr}$. For each set, 10000 Rayleigh fading channels are generated per link. Then the capacity- and Bayesian-based thresholds are computed respectively. Detection is performed using these two different thresholds. After obtaining *miss-detection* and *false-alarm* probabilities, the Bayesian cost and capacities are computed.

For partial location information case (A), when $S_{\rm cr}$ is not known, 3000 pairs $\{S_{\rm pt}, S_{\rm pr}\}$ are first generated. And for each pair, another 3000 locations for $S_{\rm cr}$ are generated according to (1) to compute the average values $\bar{L}_{\rm c}$, $\bar{I}_{\rm cp}$ and the average of a and b in thresholds (33) and (32). Then the capacities are calculated according to (8) and (34).

In cases (B) and (C) discussed in Section V, the distribution of $S_{\rm pr}$ is altered by the knowledge of $S_{\rm pt}$. In other words, we must compute the conditional distribution of $S_{\rm pr}$ given $S_{\rm pt}$. To do this, we need to use the coordinates of $S_{\rm pt}$ with respect to the absolute origin (which is $C_{\rm tx}$) instead of using its radius and angle with respect to $P_{\rm rx}$ as modeled. Hence we use ρ to specify this distance to the absolute origin, instead of r which is the radius to a specified centered point (such as $P_{\rm rx}$). To numerically compute the conditional distribution of $S_{\rm pr}$ given $S_{\rm pt}$, first, we quantize $\rho_{S_{\rm pt}}$ into 100 equal-sized bins and $\rho_{S_{\rm pr}}, \theta_{S_{\rm pt}}, \theta_{S_{\rm pr}}$ into 50 equal-sized bins within their respective ranges. Then we obtain the joint conditional probability distribution function $f(\rho_{S_{\rm pt}}, \theta_{S_{\rm pt}}|\rho_{S_{\rm pr}}, \theta_{S_{\rm pr}})$, which enables us to compute the joint distribution $f(\rho_{S_{\rm pr}}, \theta_{S_{\rm pr}}, \rho_{S_{\rm pt}}, \theta_{S_{\rm pt}})$ and eventually the cumulative distributions $F(\rho_{S_{\rm pt}}, \theta_{S_{\rm pt}})$ and $F(\rho_{S_{\rm pr}}, \theta_{S_{\rm pr}}|\rho_{S_{\rm pt}}, \theta_{S_{\rm pt}})$. With these two cumulative distribution

$$\mathcal{C} = E_{\mathbf{S},\eta}\Bigg[\lambda_1 \sum_J \prod_{i\in J}(1-p_i)\prod_{k\in\bar{J}} p_k \log\left(1+\frac{L_{\rm p}}{\sigma_z^2+\delta\sum_{i\in J} I^i_{\rm pc}}\right) + \lambda_1\delta\sum_J \prod_{i\in J}(1-p_i)\prod_{k\in\bar{J}} p_k \sum_{j\in J}\log\left(1+\frac{L^j_{\rm c}}{\sigma_z^2+I^j_{\rm cp}+\delta\sum_{l\in J,l\neq j} I^{jl}_{\rm cc}}\right) + (1-\lambda_1)\delta\sum_J\prod_{i\in J} q_i\prod_{k\in\bar{J}}(1-q_k)\sum_{j\in J}\log\left(1+\frac{L^j_{\rm c}}{\sigma_z^2+\delta\sum_{l\in J,l\neq j} I^{jl}_{\rm cc}}\right)\Bigg]. \quad (42)$$

functions, for case (B), we can first generate 3000 pairs $\{S_{\rm cr}, S_{\rm pt}\}$, then for each pair generate 3000 locations for $S_{\rm pr}$. Similarly, for case (C), we first generate 3000 locations for $S_{\rm pt}$, then for each of these locations, generate 3000 pairs $\{S_{\rm cr}, S_{\rm pr}\}$. These random generations allow us to calculate the respective average parameters in the thresholds. Then after obtaining the miss-detection and false-alarm probabilities for each set of locations, the corresponding Bayesian costs and capacities can be calculated.

In case (D) of no location information, we use the same set of locations as generated in the case of full location information, but compute only a single threshold for all the locations. Using this threshold, the Bayesian costs and capacities are computed by averaging over all locations.

2) Multiple cognitive single primary network: In this simulation, two cognitive users and one primary user are included in the network. The same radius parameters $R_{\rm c} = R_{\rm p} = R_{\rm i} = 10$ apply to both cognitive users. We only consider the cases of full location information and no location information here. A cognitive transmitter is chosen as the origin, and the locations of the primary receiver and transmitter are generated in the same way as in the single cognitive user case. The location of the second cognitive transmitter is then generated uniformly within a radius $R_{\rm i}$ around the primary receiver. The location of each cognitive receiver is generated uniformly within a radius $R_{\rm c}$ around its cognitive transmitter. (All the analysis results and simulation methods also apply if we instead choose the primary receiver to be the origin, so that the roles of all cognitive users are symmetric.) In these simulations, we set $\delta = 1$, so that either cognitive user transmits if it senses no primary transmission. Then the threshold for each cognitive user is computed using local location information as in (40) and the network sum capacity is computed as in (42).

B. Results and discussion

As shown by the simulation results in Section IV, the capacity-based threshold is more effective than the Bayesian-based threshold from a communication perspective, when evaluating based on the communication rate. In this section, we provide further comparison between these two threshold designs and also reveal the effectiveness of each design in capturing the benefit of location information, using both criteria: capacity and Bayesian cost.

1) Network throughput with capacity-based design: In Figure 4, using the capacity-based thresholds, we plot the network sum capacity, or throughput, (twice the objective (14) with $\mu = 0.5$) as a function of λ_1 with full and partial

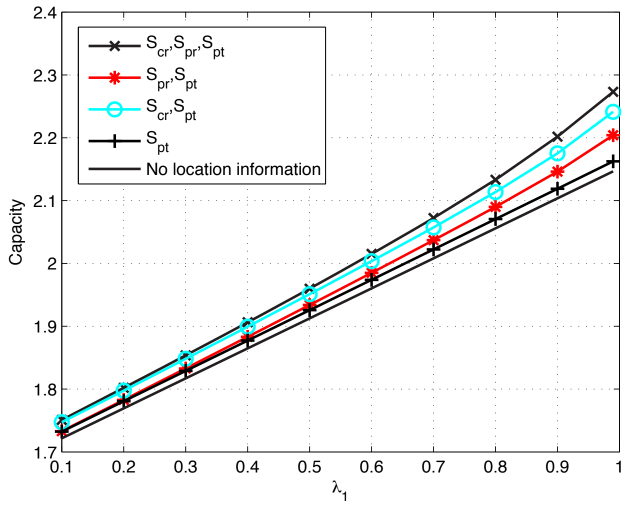

Fig. 4. Network throughput using the capacity-based threshold with full and partial location information.

location information. The plot shows that location information improves the network throughput, which agrees with intuition. Moreover, location information is more useful for network throughput as λ_1 increases. Results also show that to the cognitive Tx, information on the location of its own receiver is more beneficial than that of the primary receiver, resulting in a higher network sum capacity.

Figures 5 shows the capacities of the primary and cognitive users separately. Interestingly, more information in this case helps improve the capacity of the primary user while penalizing the capacity of the cognitive user. This impact depends on the weight μ in the capacity objective function (14) and may reverse as μ decreases, placing more emphasis on the cognitive user capacity. Also plotted in Figure 5 are the individual capacities obtained by the Bayesian-based threshold for the case of full location information. Compared to the Bayesian-based threshold, the capacity-based threshold helps to significantly boost the capacity of the cognitive user. When λ_1 approaches 1, the cognitive user's capacity with Bayesian threshold drops to near zero, while with the capacity-based threshold, the cognitive user can still maintain a significant capacity. This gain in cognitive user's capacity, however, comes at a cost to the primary user' capacity. As λ_1 increases, the primary user's capacity decreases from its maximal value with Bayesian-based threshold by up to 25%. This negative impact on the primary user's capacity can be lessened by increasing μ in the objective (14).

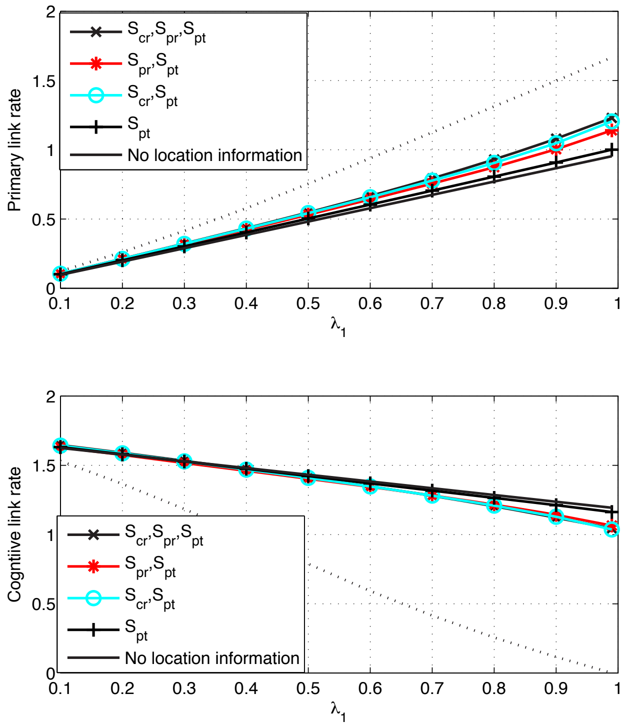

Fig. 5. Capacity of the primary user (above) and cognitive user (below) using the capacity-based threshold. Superimposed in the dotted-line for comparison is the corresponding capacity using the Bayesian-based threshold with full location information.

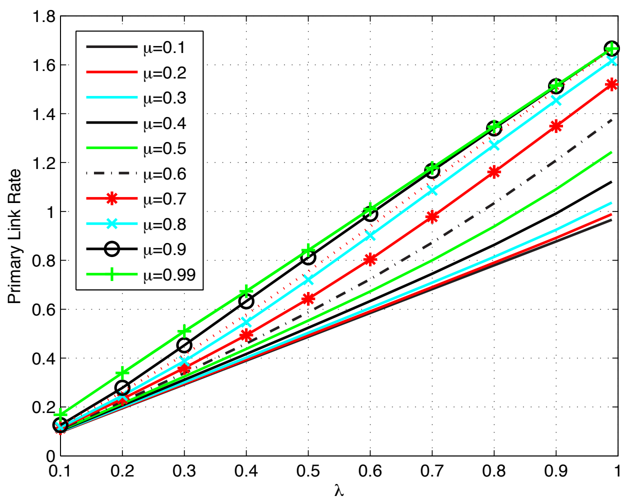

Fig. 6. Capacity of the primary user using the capacity-based threshold with different μ. Superimposed in the red dotted-line for comparison is the primary user's capacity using the Bayesian-based threshold.

2) Impact of μ on performance: Figure 6 shows the primary user's capacity with full location information for different values of μ. As expected, increasing μ helps improve the primary user's capacity. When $\mu = 0.8$, the capacity obtained with capacity-based threshold is almost the same as with Bayesian-based threshold (superimposed in the red dotted-line). When $\mu = 0.9$, capacity-based threshold supersedes

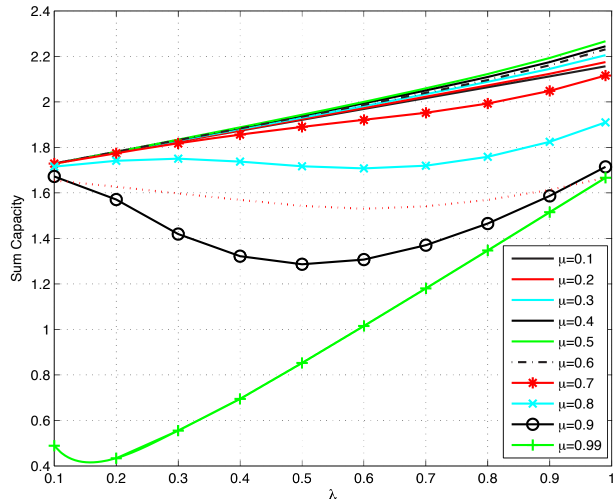

Fig. 7. Network throughput using the capacity-based threshold with different μ. Superimposed in the red dotted-line for comparison is the throughput obtained using the Bayesian-based threshold.

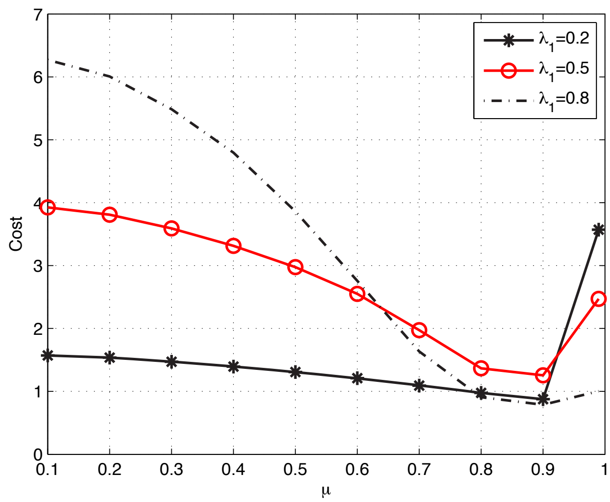

Fig. 8. Bayesian cost using the capacity-based threshold versus μ.

the Bayesian-based threshold in the resulted primary capacity, which is almost the same as that without the presence of the cognitive user (when $\mu = 0.99$). The capacity of the cognitive user understandably decreases with increasing μ, as confirmed by simulation (not shown here). Figure 7 shows the network throughput (the sum of the capacities of the primary and cognitive users) for different μ. Also superimposed in the red dotted-line is the throughput obtained with the Bayesian-based threshold, as plotted in Figure 3. The result shows that for μ as large as 0.8, the capacity-based design still achieves a higher overall throughput (compared to the Bayesian design) at a negligible impact on the primary user's capacity. Only when μ is increased to around 0.9 that the throughput suffers.

Another question is about the impact of μ on the Bayesian cost. As increasing μ improves the primary user's rate but reduces the cognitive user's rate, would it also reduce the Bayesian cost? Simulations as shown in Figure 8 confirms

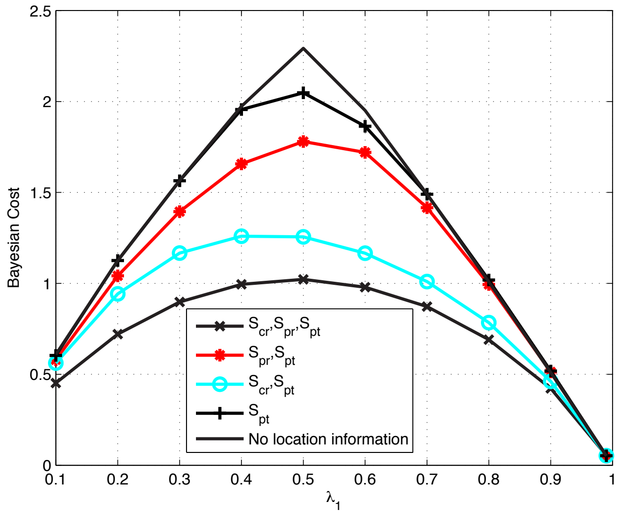

Fig. 9. Bayesian cost using the Bayesian-based threshold with full and partial location information.

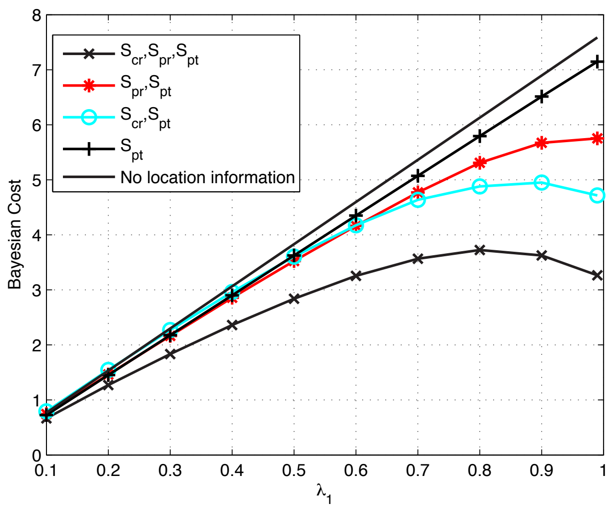

Fig. 10. Bayesian cost using the capacity-based threshold with full and partial location information.

this general trend. Interestingly, when $\mu = 0.99$ (virtually ignoring the cognitive user), the resulting Bayesian cost is actually increased (compared to $\mu = 0.9$). Thus focusing only on the primary user's capacity and ignoring the cognitive user is not a good strategy from both the network throughput and Bayesian cost point of views.

3) Bayesian cost and impact of location information: Figure 9 shows the Bayesian cost using the Bayesian-based threshold with full and partial location information. In the Bayesian cost, we see a higher impact of location information than in the network throughput. For example, compared to just knowing S_{pt}, the additional knowledge of S_{cr} can reduce the cost by 35%, while the full knowledge of location information can reduce the cost by up to 50%. Different from the network throughput in Figure 4 which gains more from location information as λ_1 approaches 1, the Bayesian cost benefits more from location information as λ_1 approaches 0.5.

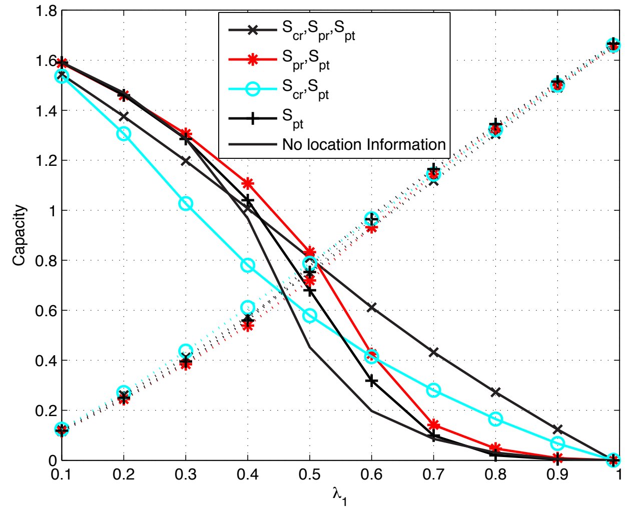

Fig. 11. Capacity using Bayesian-based threshold with full and partial location information. (Solid lines are for the cognitive link and dotted lines are for the primary link.)

4) Impact of location information when using the "other" threshold: Moreover, it is interesting to investigate the impact of location side information on each objective when using the *other* threshold. Figure 10 shows the Bayesian cost when using the capacity-optimum threshold and Figure 11 shows the two users' capacities when using the Bayesian-optimum threshold. Although the capacity-based threshold cannot minimize the sensing cost, results in Figure 10 indicate that it still captures the advantage of location information. The sensing cost is lowest with full location information and highest with partial location information of only S_{pt}. On the other hand, when using the Bayesian-based threshold, Figure 11 shows that the knowledge of location information at the cognitive transmitter has a negligible effect on the capacity of the primary user, which is a positive aspect. However, the impact of side information on the cognitive user's capacity is more complicated. Using the Bayesian-based threshold, more information does not necessarily improve the capacity of the cognitive user. Specifically in the range of $\lambda_1 < 0.5$, cases (A), (C) and (D) with partial or no information actually result in higher capacity than the case with full information. This result is against our understanding of the information principle (information never hurts [22]) and is another evidence that the Bayesian cost objective, which is an intermediate performance measure, is not as good as the capacity objective in designing a sensing threshold.

Nevertheless, the Bayesian-based threshold has an advantage in that it brings little or no loss to the primary user's capacity while also harvesting some rate for the cognitive user. In the case that the primary user pays for the spectrum usage and the cognitive user does not, the Bayesian-based threshold may be a suitable candidate since it can maintain the primary user's capacity regardless of location information available to the cognitive user. Alternatively, the capacity-based design can be used with a suitably chosen value μ.

As another observation, in all cases (A), (C) and (D), information on S_{cr} is missing. Thus the lack of knowledge

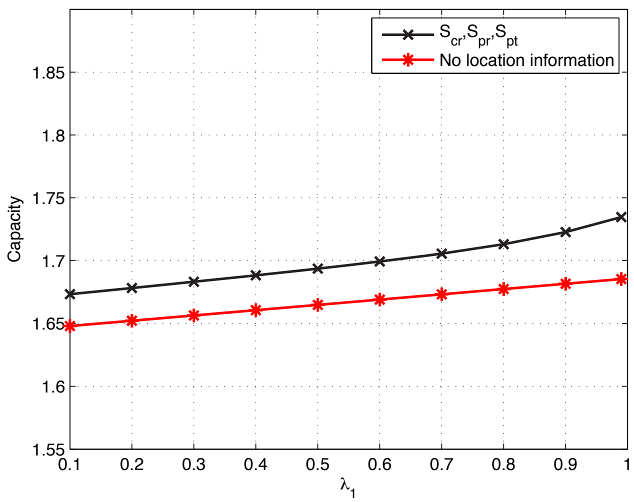

Fig. 12. Sum capacity for a network with 1 primary and 2 cognitive users using the capacity-based threshold with local location information.

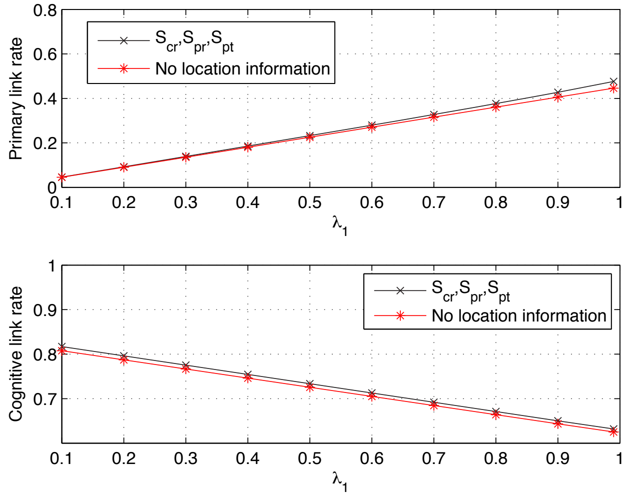

Fig. 13. Capacity of the primary user (upper) and a cognitive user (lower) in the network of 1 primary and 2 cognitive users.

about the location of its own receiver appears to have a dramatic effect on the cognitive transmitter in adjusting its Bayesian-based threshold, such that the capacity is higher than that with knowing S_{cr}. Within the group without S_{cr} knowledge, however, the cognitive user capacity increases with more location information, in line with expectation.

5) Multiple cognitive single primary network: We applied the proposed sensing threshold designs to a network with 1 primary and 2 cognitive users. Each cognitive user designs its sensing threshold independently, using only local information on the location of its receiver and of the primary transmitter and receiver. Figure 12 shows the network throughput as the sum capacity of all 3 users using the capacity-based threshold design for the two cases of full location information and no location information to each cognitive transmitter. Again we observe that just local information helps increase the network throughput. A closer look at the individual capacity of the primary and a cognitive user in Figure 13 shows that unlike the example of the single cognitive user in Figure 5, information helps increase the capacity of both primary and cognitive users in this case.

C. Some concluding remarks

In this section, we have presented two major comparisons: between two objective functions (capacity vs. Bayesian) and between different cases of location information. For the first comparison, results show that the capacity-based sensing threshold design is broadly better than the Bayesian-based design. It produces a higher network throughput and better captures the benefit of information in both measures (capacity and Bayesian cost). An intuitive reason is that the capacity objective is a communication performance measure which is more suitable for cognitive communication than a detection measure as the Bayesian objective. For the second comparison, the cases of location information, as specified earlier in Section V, are chosen based on the assumption that it is relatively easy for the cognitive users to detect the location of the primary transmitter (S_{pt}). Hence S_{pt} is included in all cases of location information (A, B and C). These cases are then differed on the knowledge of locations of the receivers (which are usually harder to detect). Results show that in most cases, as expected, more information results in better performance. Comparing between knowing the location of either receiver, it appears to be more favorable for the cognitive transmitter to know its own receiver location rather than the primary user's receiver location, as shown in Figures 4, 9 and 10.

VIII. Conclusion

We have considered energy detection spectrum sensing based on two different objectives, minimizing a Bayesian sensing cost and maximizing the network weighted sum capacity. The sensing thresholds for each objective with different cases of location side information are derived in closed-form. Results show that the Bayesian cost is much more sensitive to location information than the capacity. The Bayesian-based threshold has the advantage that it affects the primary user's capacity little in all cases of side information. But because the Bayesian sensing cost is an intermediate performance measure, the Bayesian-based threshold produces low network throughput and does not utilize the side information well to improve the transmission rates. On the other hand, the capacity-based threshold results in a much higher network throughput and captures the benefit of location side information in both the transmission rate and sensing cost measures. The impact on the individual user's capacity can be controlled by adjusting the weight in the capacity objective function. These results show that choosing a suitable objective function is critical in designing a sensing scheme.

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