Energy-Efficient Schemes for On-Demand Relaying

Fanny Parzysz
LACIME
École de Technologie Supérieure
Montreal, Canada
E-mail: fanny.parzysz@lacime.etsmtl.ca

Mai H. Vu
Dept. of Electrical and Computer Eng.
McGill University
Montreal, Canada
E-mail: mai.h.vu@mcgill.ca

François Gagnon
LACIME
École de Technologie Supérieure
Montreal, Canada
E-mail: francois.gagnon@lacime.etsmtl.ca

***Abstract*—This paper approaches the fading relay channel from an energy consumption perspective and proposes an on-demand scheme based on superposition coding and incremental redundancy (IR). It aims at minimizing the relay's energy consumption, while maintaining a rate desired by the source. The source divides its message into two parts, which allows the destination to request only the missing part(s) from the relay. We derive the set of joint power and rate allocation optimal for energy. The proposed scheme only requires local SNR knowledge and is robust to feedback delay. Compared to classical IR, this scheme always provides energy gain, which can attain 50% (at each instant and on average), and helps decrease the outage probability.**

***Index Terms*—Energy consumption; relay channel; incremental redundancy; superposition coding; capacity**

I. INTRODUCTION

Interest in cooperation for ad hoc networks has been growing in recent years, from both the theoretical capacity and the practical coding perspectives. So far, however, performance analysis of relaying systems has focused mostly on maximizing rates, outage capacity and diversity gain [1]–[4]. Given that wireless networks are often heavy-loaded and power-limited, few papers have considered sustainable rates in conjunction with energy consumption.

From a theoretical perspective, there are centralized algorithms for minimizing the total energy consumption in relay networks [5]–[7]. Although the provided gain is theoretically significant, it is mainly based on unrealistic constraints. In [7] for example, a joint power and rate allocation scheme is proposed via symbol division and flow optimization. However, the scheme remains hardly feasible in practical networks for several reasons. First, the scheme assumes sum power constraint among all nodes, instead of individual power constraint. Second, the algorithm is centralized and requires global and perfect channel state information, as well as an accurate synchronization, both of which imply a high amount of feedback. Finally, while relaying may be helpful for the source, the relay consumes extra-resources (time, power...), at the expense of not sending its own data.

On the contrary, on-demand cooperation techniques such as incremental redundancy (IR) can increase the overall network performance in terms of spectral efficiency. Responding to requests from a destination (NACK), other nodes voluntarily bring together their resources to enable or improve a transmission. Several designs based on ARQ techniques already exist, including simple answer from the relay [2], [6], [8], [9], combination of the relay and source responses [8], and advanced cooperative schemes involving Alamouti coding [1], [8]. However, these schemes are designed mainly to improve diversity, without attention to energy consumption.

In this paper, we propose an energy-efficient and practical joint power- and rate- allocation for incremental redundancy. It aims at minimizing the relay's resource consumption, while maintaining a rate desired by the source. The source message is split into two parts, allowing the destination the possibility of specifying in its NACK whether it is missing the whole message or just a part of it. Hence, in the second time slot, the relay can retransmit only the missing part(s), which helps reduce the amount of energy necessary to perform the whole transmission. We show that the proposed scheme can provide up to 50% of energy savings, in addition to decreasing outage probability. Moreover, a transmitter only needs local channel SNR knowledge. Thus the scheme can be implemented in a fully distributed manner.

The remainder of this paper is organized as follows. Section II presents the system model and a reference IR scheme. In Section III, the energy consumption is optimized with instantaneous channel knowledge. Sections IV and V analyze performance in terms of energy gain and outage probability for both cases of channel state information: with perfect SNR and with some channel knowledge errors due to feedback delay. Finally, Section VI concludes this paper.

II. SYSTEM MODEL AND OPTIMIZATION PROBLEM

A. System Model and Wireless channel

We consider a relay channel consisting of a source, a relay, and a destination. The channel gains h_i, $i \in \{1, 2, 3\}$ are zero-mean complex Gaussians with variances σ_i^2. We assume that each node can track the channel SNR variations and consider an AWGN with variance N_0 at each receiver.

Both the source and the relay are half-duplex and have the same power constraint P, within a bandwidth B. We consider a transmission over time slots of duration T. We also assume that the source sends information at a rate R (bps/Hz). From Shannon's theory, if R is below the channel capacity, we know that there exists a random code with which the error probability goes to zero. For the rest of this paper, we assume such capacity-achieving codes.

B. Reference scheme

We consider a classical two-phase IR scheme ([2], [9]) as the reference model. At the first time slot, the source sends data to the destination. If the destination cannot decode the entire message, it replies with a NACK. Without loss of generality, we can assume that this feedback link is error-less, and consumes no bandwidth and no energy. Then, if the relay is able to decode the message from the source, it transmits the whole message on to the destination at the second time slot, with full power P. Finally, the destination performs maximum ratio combining between the signals received from the source and the relay.

C. Optimization metric

To achieve energy efficiency, the effective bits-per-Joule ratio is defined as

$$\eta = \frac{\#\ \text{bits correctly decoded}}{\text{Total consumed energy}}$$

When the whole message has been successfully transmitted, an amount of RBT bits has been effectively decoded at the destination, irrespective of whether or not the relay was used. Consequently, maximizing η for successful transmissions reduces to minimizing the total consumed energy. In two cases of direct transmissions and transmissions involving the relay, the required energy is computed as

$$E_{\text{direct}} = E_{\text{source}}$$
$$E_{\text{relaying}} = E_{\text{source}} + E_{\text{relay}}$$

Since the source energy is always used, we will focus only on the case where the direct link is in outage, and where the communication (or a part of it) has to be addressed to the relay. Hence some energy savings can be obtained.

III. ENERGY-EFFICIENT INCREMENTAL REDUNDANCY

A. Proposed scheme

We propose the use of the superposition coding as depicted in Figure 1. The encoding scheme is as follows. The message m that the source wants to send is divided into two messages m_1 and m_2, with rates τR and $(1-\tau)R$ respectively, given $\tau \in [0,1]$. Message m_1 is encoded by codeword U and is allocated a power of $\lambda_s P$, with $\lambda_s \in [0,1]$. Similarly, message m_2 is encoded by another codeword V, with power $(1-\lambda_s)P$. Then U is superimposed (added) on V to form the codeword X which is sent on the channel. To decode X, the destination and the relay each performs successive interference cancellation in the best order, which depends on the set of power- and rate-allocation (λ_s, τ).

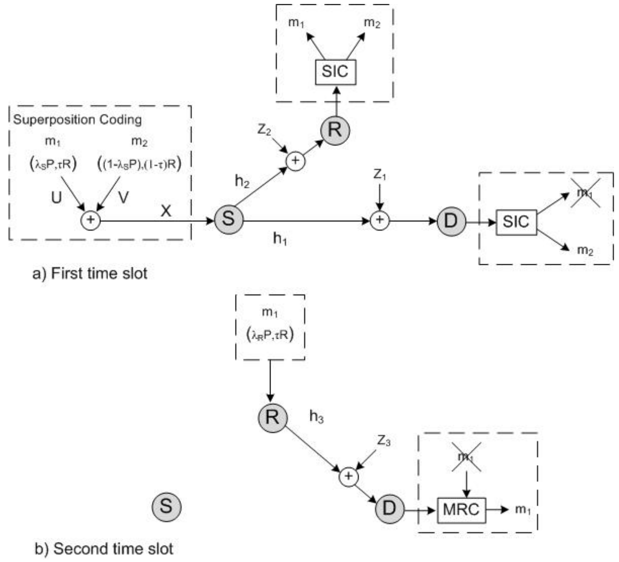

Fig. 1. Transmission scheme using superposition modulation (ex: D could not decode m_1).

If the direct link is not in outage, then the source addresses the whole message to the destination ($m_1 = 0$). Assuming now that the direct transmission is not possible, the destination sends a NACK. In this NACK, it specifies which part of the message is missing and, to further improve energy savings, the SNR received for this part in the first time slot (received directly from the source). The relay then sends this missing message again with no further splitting, but with only a portion λ_r of power. As a result, it uses the set of power and rate allocation (λ_r, τ) or $(\lambda_r, 1-\tau)$ depending on the missing part. Finally, the destination performs maximum ratio combining (MRC) for the missing part.

Without loss of generality, we can assume that the destination missed (i.e. couldn't decode) m_1. Denote $\gamma_i = \frac{P|h_i|^2}{N_0}$ and assume that the source only knows γ_1 and γ_2 from reversed-link estimation or feedback, while the relay knows γ_3 from estimation or feedback and γ_1 via the NACK sent by the destination. Unlike many resource allocation algorithms such as in [7], the source does not know the SNR on the RD-link. The proposed scheme then provides the optimal power and rate allocation set $(\lambda_s^\star,\, \lambda_r^\star,\, \tau^\star)$ that minimizes the energy consumed during the whole transmission as

$$E_T = \min_{(\lambda_s,\, \lambda_r,\, \tau) \in [0,1]^3} \left(1 + \lambda_r\right) P \tag{1}$$

where λ_r is a function of λ_s and τ (given $T = 1$).

B. Energy-optimal set $(\lambda_s^\star,\, \lambda_r^\star,\, \tau^\star)$

Proposition 1. When the direct transmission cannot be performed, and when the relay is not in outage itself in the classical IR scheme, there always exists a set of power and rate allocation (λ_s, τ) such that the destination understands m_2 and the relay understands the other part m_1. The energy-optimal set of power and rate allocation is given by

$$\lambda_s^\star\left(\gamma_1, \gamma_2\right) = \frac{1 + \dfrac{\left(1 - 2^R\right)}{\gamma_1}}{2^R - \dfrac{\gamma_2}{\gamma_1} - \gamma_2} \tag{2}$$

$$\tau^\star\left(\gamma_1, \gamma_2\right) = \frac{1}{R}\log_2\left(1 + \gamma_2 \lambda_s^\star\right) \tag{3}$$

$$\text{s.t. } \left(\lambda_s^\star, \tau^\star\right) \in [0,1]^2 \tag{4}$$

So $(\lambda_s^\star, \tau^\star)$ can be interpreted as the set giving the minimum energy such that the relay can understand m_1, while maximizing the rate of m_2 according to the remaining energy.

Proof: See Appendix A ■

This proposition ensures that positive energy gain can be obtained and that the proposed scheme can never perform worse than the reference IR scheme. Moreover, as we use MRC at the destination in the second time slot and as γ_1 is known through the NACK, the relay only uses a portion of its power such that

$$\begin{aligned} \lambda_r^\star \left(|h_1|^2, |h_2|^2\right) &= \frac{\left(2^{\tau^\star R} - 1\right)}{\gamma_3} - \lambda_s^\star \frac{\gamma_1}{\gamma_3} \\ &= \frac{\lambda_s^\star}{\gamma_3}\left(\gamma_2 - \gamma_1\right) \leq 1 \end{aligned} \tag{5}$$

This is the reason why the proposed scheme saves power (energy) compared to the reference scheme of relaying the whole message.

IV. PERFORMANCE ANALYSIS WITH PERFECT CHANNEL GAIN KNOWLEDGE

In this section, considering zero-mean complex Gaussian fading channels, we assume that the source has perfect instantaneous knowledge of SR- and SD-channel gains, and similarly, that the relay knows the RD-channel gain perfectly. For simulations, we use $B = 1$ and $P = 1$. Without loss of generality in the allocation of rate and power, R can also be set to 1.

A. *Instantaneous energy gain*

In the reference IR scheme, the relay always forwards the whole message with full power. Hence its total energy consumption is $E_{\text{Ref}} = 2P$. On the other hand, based on Proposition 1, the proposed scheme achieves an energy gain $G_{\text{inst}} = \left(E_{\text{Ref}} - E_{\text{Superposition coding}}\right)/E_{\text{Ref}}$ as:

$$G_{\text{inst}} = \frac{1}{2} - \frac{\lambda_r^\star}{2} = \frac{1}{2} - \frac{\lambda_s^\star}{2\gamma_3}\left(\gamma_2 - \gamma_1\right) \tag{6}$$

When $\gamma_1 < (2^R - 1)$ and $\gamma_1 \longrightarrow (2^R - 1)$, we can show that $\lambda_r^\star \longrightarrow 0$ and the instantaneous energy gain goes to 50%. While using a classical IR scheme always leads to a full use of the relay, the proposed scheme approaches the energy consumption of the direct transmission : the missing part of the message is so small that the relay goes on to consume almost no energy at all. Moreover, even in the case of minimal instantaneous gain is minimal when $\gamma_1 \longrightarrow 0$, which means that $(\lambda_s^\star, \tau^\star) = (1, 1)$, the energy gain $G_{\text{inst}} \longrightarrow \frac{1}{2} - \frac{\gamma_2}{2\gamma_3} = \frac{1}{2} - \frac{2^R - 1}{2\gamma_3}$ is strictly positive, because of the channel knowledge at the relay. Figure2 illustrates the energy gain versus the channel gains..

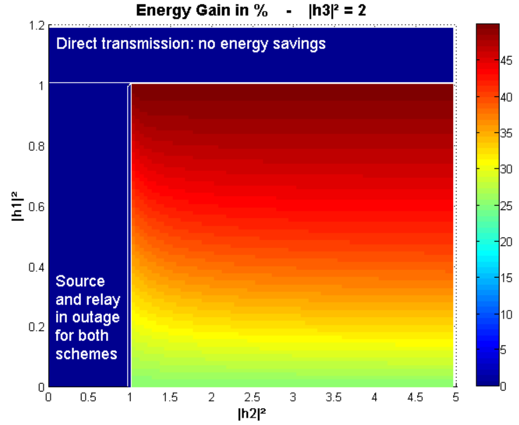

Fig. 2. Instantaneous energy savings for different SD- and SR- gains

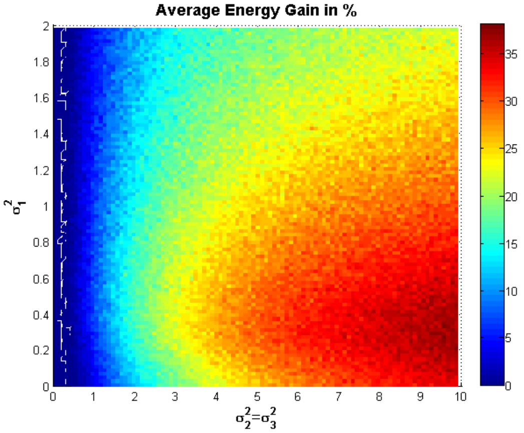

Fig. 3. Average energy gain for different channel variances

B. *Average energy gain*

Now we evaluate the average energy savings. The proposed scheme provides energy gain compared to the reference IR scheme when direct transmission cannot be performed and the relay is not in outage itself. We can then evaluate the average gain as

$$G_{\text{ave}} = \frac{1}{2} - \frac{E\left[\lambda_r^\star\right]}{2} \tag{7}$$

This is the maximum energy gain that can be obtained, while maintaining the rate R desired by the source at each instant. Figure 3 illustrates the average energy savings for different channel variances. As expected, when the direct channel variance σ_1^2 is low, the SD-link is often in outage. Then, a large variance on the relaying path provides high energy gain. The gain goes to 50% again when the SR-channel variance becomes very high. Therefore, the proposed scheme takes advantage of channel variations and performs in an opportunistic manner.

C. *Outage probability*

We have so far only focused on the cases where the relaying path is not in outage in the reference IR scheme. However, this outage region is not equivalent to that in the proposed scheme. Indeed, for the reference scheme, we have the outage as

$$P_{\text{out,ref}} = \mathrm{P}\left[|h_3|^2 + |h_1|^2 \leq \frac{(2^R - 1)N_0}{P}\right]$$

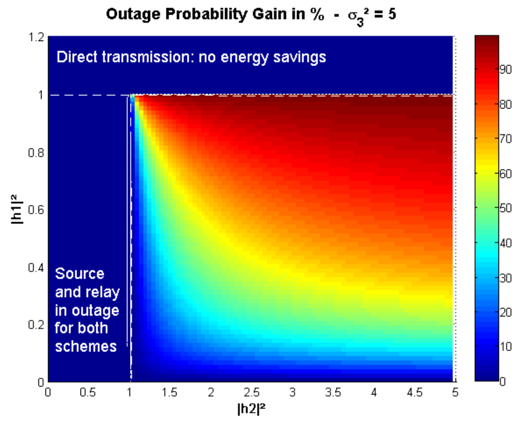

Fig. 4. Outage probability gain given $|h_1|^2$ and $|h_2|^2$

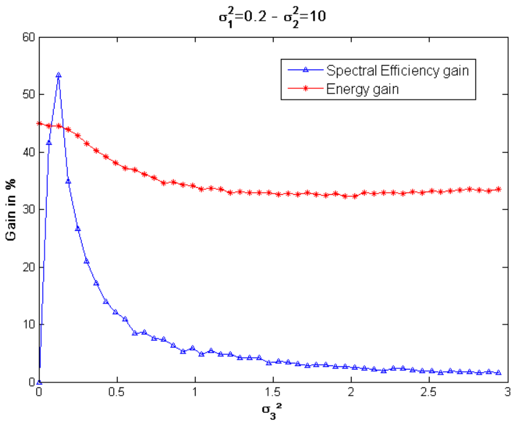

Fig. 5. Spectral efficiency and energy gains

On the contrary, in the proposed scheme, outage only occurs if $\tau^\star R \leq \log_2\left(1+\frac{P\left(|h_1|^2\lambda_s^\star+|h_3|^2\right)}{N_0}\right)$, when the maximum transmit power P is used to enable the relay-assisted transmission ($\lambda_r^\star = 1$). This leads to:

$$P_{\text{out,new}} = \mathrm{P}\left[|h_3|^2 + \lambda_s^\star |h_1|^2 \leq \frac{(2^{\tau^\star R}-1)N_0}{P}\right]$$

Therefore, given $|h_1|^2$ and $|h_2|^2$, the outage probability of the proposed scheme is decreased by a factor as

$$\frac{P_{\text{out,new}}}{P_{\text{out,ref}}} = \exp\left(-\frac{\left(2^R - 2^{\tau^\star R}\right)\frac{N_0}{P} - |h_1|^2\left(1-\lambda_s^\star\right)}{\sigma_3^2}\right) \quad (8)$$

Figure 4 illustrates the outage gain, defined as $(P_{\text{out,ref}} - P_{\text{out,new}})/P_{\text{out,ref}}$, for different values of given $|h_1|^2$ and $|h_2|^2$ and for $h_3 \sim \mathcal{CN}(0, \sigma_3^2)$.

This result again shows the opportunistic advantage of the proposed scheme. The more the RD-link varies, the greater the relative reduction in outage probability is. Moreover, when $\gamma_1 < (2^R - 1)$ and $\gamma_1 \longrightarrow (2^R - 1)$, the outage probability of the proposed scheme decreases to 0: as $\lambda_r^\star \longrightarrow 0$, the RD-link is never in outage, regardless of the link quality.

D. Spectral Efficiency

Spectral efficiency is another performance measure of interest, and it can be derived from the above analysis of outage probability. The spectral efficiency of the reference incremental redundancy $\overline{R}$ is given in Eq. (34) of [2]. For the proposed scheme, it can be computed as follows

$$\begin{aligned}\overline{R}^\star =& \frac{R}{2}.\mathrm{P}\left[\text{D: } (m_1,m_2)\right] + R.\mathrm{P}\left[\text{R: } (m_1,m_2)\right] \\ &+ R.\mathrm{P}\left[\text{D: } m_2,\ \text{R: } m_1\right]\end{aligned} \quad (9)$$

where $\mathrm{P}\left[\text{D: } m_2,\ \text{R: } m_1\right]$ is the probability that the destination decodes m_2 from the direct transmission and m_1 from the transmission involving the relay.

Figure 5 plots the average energy savings and the spectral efficiency gain $\frac{\overline{R}^\star}{\overline{R}} - 1$ that are obtained through the proposed relaying strategy. The spectral efficiency is significantly improved, especially when the RD-link is weak. Indeed, while a classical IR scheme suffers from high outage, the minimization of the relay's response allows the transmission to succeed in the proposed scheme. Moreover, energy savings remains noticeable.

V. PERFORMANCE ANALYSIS WITH IMPERFECT CHANNEL KNOWLEDGE

The in previous section assumes perfect transmit channel information (CSIT). If such CSIT is obtained by estimation over the reversed channel or feedback, a delay may exist between the channel measurement and its effective realization. In this section, we show that the proposed strategy can easily overcome potential outdated measurements and still provides significant energy gain.

A. Channel model

To model such a channel acquisition, we adapt the dynamic CSIT model proposed in [10]. An initial channel measurement $h_{i,0}$ at time 0, assumed correct, is actually received by the transmitter at time t due to feedback delay. The temporal correlation between $h_{i,0}$ and $h_{i,t}$ is modelled by $\rho \in [0,1]$. This parameter then measures the channel estimation quality, from perfect channel knowledge (no delay, $\rho = 1$) to channel long-term statistics including mean and variance ($\rho = 0$). Thus, a channel realization $h_{i,t}$ can be expressed as

$$h_{i,t} = \hat{h}_{i,t} + e_{i,t} \quad (10)$$

where $\hat{h}_{i,t}$ is the estimated value. The error $e_{i,t}$ is modelled as a complex Gaussian stationary random process. Using a simplified model, we adapt Eq.(11) of [10] and express the estimated channel and error variance as:

$$\begin{aligned}\hat{h}_{i,t} &= \rho h_{i,0} \\ \sigma_{e,i}^2 &= \left(1-\rho^2\right)\sigma_i^2\end{aligned} \quad (11)$$

Here, ρ is usually above 0.7 in practical systems, such that the correlated energy is more than $\frac{1}{2}$. Otherwise, feedback is deemed to be insufficiently accurate to be helpful.

B. Optimization problem

When the optimal set $(\lambda_s^\star, \lambda_r^\star, \tau^\star)$ depends on exact channel values $\{h_{i,t}\}_{i\in\{1,2,3\}}$ as in previous sections, outage is only due to deep fades. This is no longer the case with feedback delay, as channel knowledge errors also causes outage and should be taken into account. Given estimated channel gains, a set $(\lambda_s, \lambda_r, \tau)$ is able to successfully convey the two parts of the message only with a certain probability. Compared to the perfect CSIT case, we can upper-bound the outage probability increase by $P_{e_1} + P_{e_2} + P_{e_3}$, where $P_{e_i} = \mathrm{P}\left[|h_i|^2 < |\hat{h_i}|^2 \text{ given } |\hat{h_i}|^2\right]$: an outage occurs when the effective channel gain is below the estimated one.

We now redefine the optimization problem. To enhance energy savings, we allow a small outage probability increase, and define a tolerance threshold ε and a new constraint as

$$P_{e_i} \leq \varepsilon, \quad i \in \{1,2,3\} \tag{12}$$

We then maximize the expected energy gain, given estimated channel values as

$$\begin{aligned} G^\star_{\text{error}}\left(|\hat{h}_1|^2, |\hat{h}_2|^2, |\hat{h}_3|^2\right) &= \max_{(\lambda_s, \lambda_r, \tau)} \mathrm{E}\left[\frac{1}{2} - \frac{\lambda_r}{2}\right] \\ &\text{s.t. } P_{e_i} \leq \varepsilon, \quad i \in \{1,2,3\} \end{aligned} \tag{13}$$

where the expectation is over both fading and CSIT error.

C. Suboptimal sets $\left(\tilde{\lambda}_s, \tilde{\lambda}_r, \tilde{\tau}\right)$

Solving the optimum in (13) is quite challenging. A suboptimal set $\left(\tilde{\lambda}_s, \tilde{\lambda}_r, \tilde{\tau}\right)$ for problem (13) can be derived based on the previous perfect channel knowledge case. It consists in voluntarily lowering the estimated channel gain until condition (12) is satisfied, and choosing the rate and power allocation set according to those lowered channel gains. We have the energy gain as

$$G^\star_{\text{sub}}\left(|\tilde{h}_1|^2, |\tilde{h}_2|^2, |\tilde{h}_3|^2\right) = \mathrm{E}\left[\frac{1}{2} - \frac{\tilde{\lambda}_r}{2}\right] \tag{14}$$

$$\begin{aligned} \text{where } \tilde{\lambda}_r &= \min_{(\lambda_s\ \tau)} \lambda_r \\ &\text{s.t. } P_{e_i} \leq \varepsilon, \quad i \in \{1,2,3\} \end{aligned} \tag{15}$$

Here, $|\tilde{h_i}|^2$ is the lowered channel gain, that is $|\tilde{h_i}|^2 \leq |\hat{h_i}|^2$ so that (12) is satisfied.

Proposition 2. The energy-suboptimal set of power and rate allocation that solves (14) is given by:

$$\tilde{\lambda}_s = \frac{1 + \dfrac{(1-2^R)}{\tilde{\gamma}_1}}{2^R - \dfrac{\tilde{\gamma}_2}{\tilde{\gamma}_1} - \tilde{\gamma}_2}$$

$$\tilde{\tau} = \frac{1}{R}\log_2\left(1 + \tilde{\gamma}_2.\tilde{\lambda}_s\right)$$

$$\tilde{\lambda}_r = \frac{\tilde{\lambda}_s}{\tilde{\gamma}_3}\left(\tilde{\gamma}_2 - \tilde{\gamma}_1\right)$$

where $\tilde{\gamma}_i = \max\left[\frac{P|\tilde{h}_i|^2}{N_0}\right]$ subject to (12) and $|\tilde{h_i}|^2 \leq |\hat{h_i}|^2$.

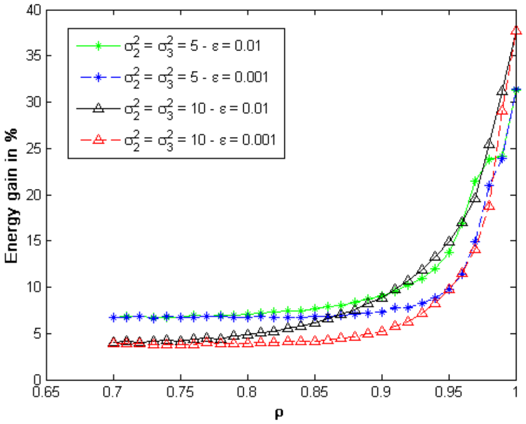

Fig. 6. Average energy gain for various temporal channel correlation

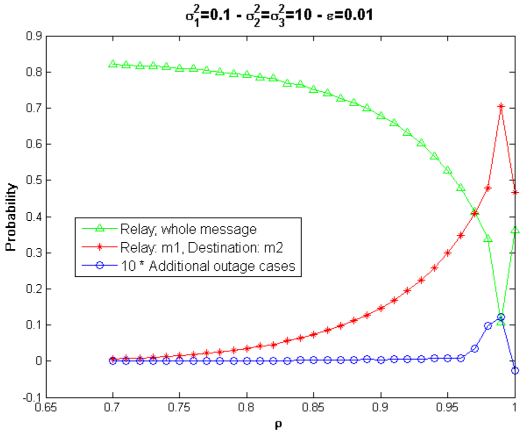

Fig. 7. Relay behaviour for various temporal channel correlation

D. Simulation results

Simulations results show that the proposed relaying scheme presents two distinct behaviours, depending on the feedback delay (or equivalently the temporal correlation ρ), and leads to different energy gains and outage reduction. Figures 6 and 7 respectively plot the average energy gain $\mathrm{E}[G_{\text{sub}}]$ and the number of additional outage cases (in percentage). Both figures are evaluated by comparing the reference IR scheme to the suboptimal allocation set of Proposition 2. The lowered channel SNRs $(\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3)$ are found numerically for each estimated channel gain. These results bring the following observations:

1) High feedback delay: From Proposition 2, when channel measurements are outdated ($\rho \in [0.7; 0.85]$), the energy gain is affected the most by error variances $\{\sigma^2_{e,2}, \sigma^2_{e,3}\}$. When the variances become significant, the estimated channel gains have to be lowered substantially such that (12) is satisfied. In this case, the lowered channel gains approach zero, and the proposed scheme naturally converges to classical incremental redundancy. Therefore, the whole message is sent to the relay, and no additional outage case (due to feedback delay) occurs, as shown in Figure 7. However, unlike the reference scheme, feedback allows the relay to adjust its transmit power to the RD-link quality, and an energy gain can nevertheless be obtained.

2) Low feedback delay: When the feedback accuracy is high ($\rho \in [0.85; 1]$), the outage tolerance threshold ε has the most impact on energy gain. As depicted in Figure 7, the proposed scheme performs message division, and by taking advantage of the allowed outage loss, we obtain significant energy savings. Consequently, the proposed scheme suffers from slight spectral efficiency loss, which is bounded by 3ε.

Again this relaying strategy performs in an opportunistic manner, and benefits from high channel variations, although this likewise increases channel estimation error variances. This is because when channel variances increase, the estimated gains $\hat{h}_{i,t}$ are more likely to be high, which sufficiently compensates for the gain lowering.

Even tough the proposed resource allocation set $(\tilde{\lambda}_s,\, \tilde{\lambda}_r,\, \tilde{\tau})$ is suboptimal for (13), it establishes a simple, but already noticeable, lower-bound on the energy savings that can be obtained through the proposed relaying strategy.

VI. CONCLUSION

We tackle the issue of energy consumption in an on-demand cooperation system and design an efficient joint rate and power allocation scheme for incremental redundancy. Applying superposition coding, the source divides its message into two parts. One is directly decoded by the destination and the other is conveyed by the relay. The source allocates a portion of the total rate and power to each, so as to maximize the part intended for the destination and hence minimize the relay's energy consumption. As this joint optimization is based only on local SNR knowledge, the proposed scheme can be implemented in a fully distributed manner. It is also robust to potential outdated channel state information.

Analysis and simulations both show that the performance of the proposed scheme is lower-bounded by classical IR performance. Moreover, up to 50% of energy savings and significant outage decrease can be obtained. As a result, this paper shows that relaying can be energy-efficient under realistic constraints, while sustaining a desired rate.

APPENDIX

Proof of Proposition 1

When the direct transmission cannot be performed, and when the relay is not in outage itself in the classical IR scheme, the following bounds are satisfied:

$$\log_2(1+\frac{P|h_1|^2}{N_0}) = C_{\text{SD}} < R \leq C_{\text{SR}} = \log_2(1+\frac{P|h_2|^2}{N_0})$$

First, let's consider the set of all $(\lambda_s, \tau) \in [0,1]^2$ that satisfy:

$$(1-\tau)R \leq \log_2\left(1+\frac{P(1-\lambda_s)|h_1|^2}{P\lambda_s|h_1|^2+N_0}\right) < C_{\text{SD}} \qquad (16)$$

This set is non-empty as long as $|h_1|^2 \neq 0$. It refers to all power and rate allocation sets that allow the destination to decode m_2, given that m_1 creates interference.

Second, since the relay is not in outage itself in the classical IR scheme, we have $|h_1|^2 \leq |h_2|^2$ and it immediately follows that the relay can also decode m_2.

Third, let's consider the set of all $(\lambda_s, \tau) \in [0,1]^2$ satisfying both (16) and

$$\tau R \leq \log_2\left(1+\frac{\lambda_s P|h_2|^2}{N_0}\right) \leq C_{SR} \qquad (17)$$

such that the relay can also decode m_1. We can show that (16) and (17) are not in contradiction.

Therefore, there always exists an eligible rate and power allocation set (λ_s, τ) for the problem (1), and this set satisfies

$$\tau \leq \frac{1}{R}\log_2\left(1+\frac{P\lambda_s|h_2|^2}{N_0}\right) \qquad (18)$$

$$\tau \geq 1-\frac{1}{R}\log_2\left(1+\frac{P(1-\lambda_s)|h_1|^2}{P\lambda_s|h_1|^2+N_0}\right) \qquad (19)$$

Analysis shows that minimizing the total energy consumption in (1) leads to set λ_s at the intersection of the upper-bound (18) and the lower-bound (19).

REFERENCES

[1] C. Zhang, H. Yin, W. Wang, and W. Guo, "ARQ protocols for two-user cooperative diversity systems," in *Wireless Communications, Networking and Mobile Computing, 2008. WiCOM '08. 4th International Conference on*, oct. 2008, pp. 1 –6.

[2] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *Information Theory, IEEE Transactions on*, vol. 50, no. 12, pp. 3062 – 3080, 2004.

[3] T. M. Cover and J. A. Thomas, *Elements of information theory*. New York : J. Wiley, 1991.

[4] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. Cambridge University Press, 2005.

[5] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *Information Theory, IEEE Transactions on*, vol. 51, no. 6, pp. 2020–2040, 2005.

[6] F. S. Tabataba, P. Sadeghi, and M. R. Pakravan, "A new power allocation scheme for amplify and forward incremental relaying," in *Signal Processing Advances in Wireless Communications, 2009. SPAWC '09. IEEE 10th Workshop on*, 2009, pp. 494–498.

[7] T. F. Wong, T. M. Lok, and J. M. Shea, "Half-duplex cooperative transmission for the relay channel with flow optimization," in *Information Sciences and Systems, 2007. CISS '07. 41st Annual Conference on*, 2007, pp. 402–407.

[8] G. Yu, Z. Zhang, and P. Qiu, "Cooperative ARQ in wireless networks: Protocols description and performance analysis," in *Communications, 2006. ICC '06. IEEE International Conference on*, vol. 8, 2006, pp. 3608 –3614.

[9] T. Renk, H. Jakel, F. K. Jondral, D. Gunduz, and A. Goldsmith, "Outage capacity of incremental relaying at low signal-to-noise ratios," in *Vehicular Technology Conference Fall (VTC 2009-Fall), 2009 IEEE 70th*, 2009, pp. 1–5.

[10] M. Vu and A. Paulraj, "On the capacity of MIMO wireless channels with dynamic CSIT," *Selected Areas in Communications, IEEE Journal on*, vol. 25, no. 7, pp. 1269–1283, 2007.