

# An Overview of Scaling Laws in Ad Hoc and Cognitive Radio Networks \*

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**Abstract.** Currently, wireless communications are changing along the lines of three main thrusts. The first is the introduction of secondary spectrum licensing (SSL). Regulations on the usage of licensed spectra are being loosened, encouraging unused primary spectrum to be licensed, often in an opportunistic manner, to *secondary* devices. The second is the introduction of *cognitive radios*. These wireless devices are able to sense and adapt in a “smart” manner to their wireless environment, making them prime candidates to becoming secondary users in SSL initiatives. Finally, as we approach the communication limits of point-to-point channels, and as wireless devices become cheap and ubiquitous, the focus is shifting from single to multiple communication links, or *networks*. In this paper, we provide an overview of the recently established theoretical limits, in the form of sum-rates, or throughput, of two main types of networks: ad hoc networks, in which the devices are homogeneous, and cognitive networks, in which a mixture of primary and secondary (or cognitive) devices are present. We summarize and provide intuition on how the throughput of a network scales with its number of nodes  $n$ , as  $n \rightarrow \infty$ , under different network and node capability assumptions.

**Keywords:** Scaling laws, ad-hoc networks, cognitive networks, cognitive radio.

## 1. Introduction

### 1.1. SECONDARY SPECTRUM OPPORTUNITIES

As wireless users and applications demand ever more bandwidth, efficient usage of the licensed wireless spectra is becoming a necessity. In spite of this, recent measurements [5] have shown that large portions of prime licensed spectrum remain unused a significant percentage of the time. As a remedy, regulatory bodies such as the Federal Communications Commission in the US and the European Commission’s Radio Spectrum Policy Group in the EU, are proposing the secondary usage of this spectrum. In contrast to licensed bands, to which entities such as TV stations or cellular operators are granted exclusive access, new regulations would allow for devices which are able to sense and adapt to their spectral environment, such as cognitive radios, to become *secondary* or *cognitive users*. These cognitive users opportunistically

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\* Portions of Section 4 has appeared in Vu et al., 2007 [16].



employ the spectrum of the *primary users* in a way unharmful to these primary users. Primary users generally associate with the primary spectral license holder, and thus have a higher priority right to the spectrum.

## 1.2. COGNITIVE RADIOS: EXPLOITING SECONDARY SPECTRUM USAGE

In a parallel thrust, *software defined radio* technologies, or radios in which operating parameters may be set by software rather than in hardware alone, are promising great gains in terms of flexibility, cost, and time to market. Such devices have the ability to transmit and receive using a variety of protocols and modulation schemes, which are enabled through reconfigurable software. *Cognitive radio*, a term coined by Mitola [13], takes software defined radio to a new level: these “smart” radios sense their RF environment and are able to adapt their transmission parameters independently according to the local regulations, quality of service requirements, or the sensed spectral activity. One of the many uses for cognitive radio is secondary spectrum usage.

The simplest model for secondary spectrum usage is one in which a single cognitive source and destination pair, or link, wishes to share the spectrum with an existing primary source and destination pair. Classical secondary spectrum access approaches involve sharing the channel in an orthogonal manner: the cognitive radio would sense an empty time (TDMA) or frequency (FDMA) or even code (CDMA) and transmit over some of these slots. Interference is thus avoided. However, recent work has exploited the smart nature of the cognitive secondary link to improve upon the spectral efficiency achieved by these orthogonal schemes. In the “cognitive radio channel”<sup>1</sup> [4], the cognitive secondary user is assumed to be able to obtain the message of the primary user. This assumption is motivated by the cognitive capabilities of the secondary node. The primary and secondary users proceed to transmit simultaneously, in a non-orthogonal fashion. However, because the secondary transmitter knows the primary’s message, it may trade off between boosting this message (relaying it) and mitigating its interfering effect on the secondary receiver. Rate regions for the most general cognitive radio channel model are obtained in [4, 12, 18, 8], while the capacity region for the cognitive radio channel in Gaussian noise, under certain weak interference conditions, was obtained in [10]. The rate regions obtained show significant spectral efficiency gains over the traditional orthogonal spectrum sharing techniques, and encompass these schemes as a limiting case. These gains are results of the cognitive

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<sup>1</sup> The cognitive radio channel is also called an “interference channel with degraded message sets” [10, 18] or a channel with a cognitive transmitter [12]

transmitter intelligently aiding, as well as mitigating interference from the primary user.

In the above simple cognitive radio channel consisting of 2 transmitters and 2 receivers, cognition was exploited on the secondary link to improve the rates of both primary and secondary users. A question that naturally arises is how such rates scale in a large network of devices. The rest of this article is dedicated to exploring a fundamental limit – the scaling law – of the sum-rate of networks of wireless devices, in both non-cognitive and cognitive settings.

### 1.3. AD HOC AND COGNITIVE NETWORKS

As single-link wireless technologies mature with accelerated understanding over the last decade, attention now turns to how these devices perform in a network. Of great interest is the infrastructure-less network in which devices can communicate in an ad hoc manner. Applications of these networks abound, for example, mobile IP networks, smart home devices, spontaneously formed disaster recovery or military networks, and dispersed sensor networks. Ad hoc communications become even more relevant in a cognitive setting, as the secondary devices are opportunistic in nature and hence are likely to operate in an ad hoc fashion.

While we understand precisely the capacity of a point-to-point link and can design codes to attain this capacity closely, the capacity of a network remains undefined. Multiple dimensions play a role in the capacity of a network: the number of nodes in the network, the node density, the network area, the power and rate of each node. As nodes can join the network randomly and the network size can grow large, what throughput the network can sustain as more nodes join is of particular interest. In other words, how the network sum-rate, or equivalently, the per-node throughput, scales with the number of nodes.

In this paper, we provide an overview of recent *fundamental* and *theoretical* developments in wireless ad hoc and cognitive networks. We present different scenarios in which  $n$  wireless transceivers are placed in a network and ask the question:

*How does the sum-rate of the network scale with the number of nodes,  $n$ ?*

We specifically contrast the cases in which all nodes are primary nodes (the ad-hoc network case) and those in which a mixture of primary and secondary nodes may be employed (the cognitive network case). Tools from *information theory* are used in obtaining the sum-rate scaling results. The rates referred to throughout this article will be *achievable rates* [3], rates that may be achieved with asymptotically small error probability.

The scaling law of a network is usually established based on lower and upper bounds to the sum rate. Lower bounds are obtained by suggesting a specific transmission strategy, which provides an achievable rate. Upper bounds can be obtained by theoretically or by relaxing certain assumptions. When scaling law of the lower bounds meets that of the upper bound, then we have the precise sum-rate scaling law of the network.

For *ad hoc networks*, in which  $n$  nodes are homogeneous and are located randomly, the scaling law depends greatly on the node distribution and the physical-layer processing capability, more specifically the ability to cooperate among nodes. In the interference-limited regime, in which no cooperation is allowed (except simple forwarding) and all nodes treat other signals as interference, the per-node throughput (which equals the sum rate divided by  $n$ ) scales at most as  $1/\sqrt{n}$  [7]. If the nodes are uniformly distributed, a simple nearest-neighbor forwarding scheme achieves only  $1/(n \log(n))$  per-node throughput [7]. When the nodes are distributed according to a Poisson point process, however, a backbone-based routing scheme achieves the per-node scaling of  $1/\sqrt{n}$  [6], meeting the upper bound. The scaling law of an interference-limited ad hoc network therefore is  $1/\sqrt{n}$  for the per-node throughput.

On the other hand, when nodes are able to cooperate, a much different scaling law emerges. Upper bounds based on the max-flow min-cut bound ([19, 20], [15], [9, 2]) as well as MIMO techniques ([11, 14]) have been analyzed, among which a simple bound shows that the sum rate can grow no faster than  $n \log(n)$  [15]. Specifically, a hierarchical scheme can achieve a *linear* grow in the sum rate, corresponding to a constant per-node throughput [15]. Here nodes form clusters; nodes within a cluster exchange information and then cooperate to communicate to nodes in another cluster. This cluster formation may be layered (clusters of clusters), forming a hierarchical scheme in which eventually all nodes will be able to cooperate in a MIMO fashion. Cooperative schemes therefore can provide significantly higher throughput than the non-cooperative ones.

While ad hoc networks assume a form of node symmetry: all nodes have similar capabilities and of equal importance, in *cognitive networks* which employ primary and secondary nodes, this symmetry breaks down. The primary nodes usually have higher priority access to the spectrum. The cognitive nodes, on the other hand, may need to sense their environment and operate on an opportunistic base. This leads us to an initial study of a simple cognitive network, in which  $n$  cognitive devices are sharing the spectrum with one primary user. While ensuring an outage condition from the primary user, the cognitive users also try to sustain their rates. Assuming a bounded transmitter-receiver

distance and using *single-hop* transmission, these cognitive users can achieve a *linear* growth in the sum rate [16]. Without cooperation, this linear growth rate is in sharp contrast to the interference-limited ad hoc network. The key difference here is the single-hop transmission, which appears more suitable in the secondary, opportunistic setting of a cognitive network.

#### 1.4. PAPER OUTLINE

This work is structured as follows. In Section 2, we describe the channel and network models that are most commonly used to derive scaling law results. In Section 3, we discuss current results on the scaling laws of traditional ad hoc networks. In Section 4, we describe the recently introduced *cognitive networks*. In Section 5 we conclude.

## 2. Network and Channel Models

### 2.1. NETWORK MODEL

Consider  $n$  pairs of devices wishing to communicate between each other. Each pair consists of a single transmitter communicating with a single receiver, located on a two-dimensional plane. This setup excludes multiple-access, broadcast and relaying types of communication. These pairs of devices are also referred to as “users” interchangeably. There are two common network models: dense networks and extended networks. In dense networks, the network area stays constant as the number of users grows. In extended networks, the area grows linearly with the number of users, provided the user density remains constant. A connection exists between these two models, and results for one can be transferred to the other with an appropriate scaling. Most results presented in article are for the extended network setting with a constant node density and an area growing with  $n$ .

A figure of merit is the total network capacity (also called the sum rate or the throughput). This sum rate is defined as

$$C(n) = \sum_{i=1}^n R_i \quad (\text{bits/channel use}), \quad (1)$$

where  $n$  is the number of users and  $R_i$  is the information rate of user  $i$ . A related figure is the capacity per user

$$R(n) = \frac{1}{n}C(n) \quad (\text{bits/channel use, per user}). \quad (2)$$

The main question we consider is how the capacity  $C(n)$  (equivalently  $R(n)$ ) scales as the number of users  $n$  grows to infinity.

In a network, different assumptions on the capabilities of the physical-layer signal processing lead to different scaling laws. In particular, the ability of the nodes to cooperate greatly affects the scaling of the throughput. At one extreme, the interference-limited regime, each receive node treats all signals other than its intended signal as noise. At the other extreme, full cooperation is allowed among the nodes, at the expense of information exchanging. Just as node capabilities affect the scaling laws, so does the spatial user distribution. We will see example of two different user distributions and their corresponding scaling rates.

## 2.2. CHANNEL MODEL

The wireless propagation channel typically includes path loss with distance, shadowing and fading effects. Given a transmitter-receiver distance  $d$ , the channel is given as

$$h(d) = \frac{e^{-\gamma d}}{d^{\alpha/2}} h_s \quad (3)$$

where

- $d$  the transmitter-receiver distance
- $\alpha$  power path loss exponent
- $\gamma$  absorption constant ( $\gamma \geq 0$ )
- $h_s$  shadowing and fading component

Most analyses so far, however, either omit the fading component ( $h_s = 1$ ) or include it in a simple form as a uniform random phase [20, 15]. In the following sections, unless explicitly stated, we will assume that  $h_s = 1$  and  $\gamma = 0$ . In other words, we consider channels with path loss alone. The results therefore are applicable to *large-scale* networks. Furthermore, we focus on the case that  $\alpha > 2$ , which is often found in practice. We also assume that each user within a network transmits with equal power, although users of different networks may transmit with different powers.

## 3. Ad Hoc Networks

### 3.1. INTERFERENCE-LIMITED NETWORKS WITH UNIFORM NODES

The ad hoc network with *uniformly distributed nodes* was first studied in the seminal paper [7]. In this network, the nodes are randomly and uniformly distributed, with *random pairing* of transmitter and receiver. The result applies to the simple path-loss model in (3) with  $\alpha > 2$ . To establish a lower bound on the sum rate, the signal model assumes a threshold on the received SINR of each user as

$$\frac{P/d_i^\alpha}{\sigma_n^2 + \sum_{i \neq k} P/d_{ik}^\alpha} \geq \beta \quad (4)$$

Here  $\sigma_n^2$  is the noise power,  $d_k$  is the distance between the transmitter and receiver of user  $k$ ,  $d_{ik}$  is the distance between the receiving node  $k$  and the interfering node  $i$ . This condition corresponds to an *exclusion region* around the receiver of each user, such that these regions of receivers on the same frequency band are disjoint [21]. Then for nodes that are *uniformly and randomly distributed* within a unit square (corresponding to a dense network), the per-node transmission rate is lower bounded as [7]

$$R(n) \geq \frac{c(\beta)}{\sqrt{n \log n}}, \quad (5)$$

where  $c(\beta)$  is a constant dependent on  $\beta$ . (It is proportional to  $\beta^{-1/2\alpha}$  and hence increases to  $\infty$  as  $\beta$  approaches 0.) This lower bound is achieved by *nearest-neighbor forwarding* in a multihop routing scheme. The network is divided into grids, and communication is carried out in a multihop fashion, in which nodes keep forwarding the information to the neighboring cell nearest to the straight line connecting the transmitter and the receiver.

An upper bound can be obtained without the exclusion region, nor the interference constraint (4), as in [1]

$$R(n) < \frac{c_1}{\sqrt{n}}, \quad (6)$$

where  $c_1$  is a constant. This result relaxes further the power assumption such that each transmitter can have its own power. This upper bound applies to any interference condition, without requiring a hard threshold on the SINR. That is, even if nodes try to communicate when its SINR is below a certain level  $\beta$ , in addition to the communication established above this level, the per-user rate is still upper-bounded by the order  $1/\sqrt{n}$ .

The above bounds on the scaling law, although originally developed for dense networks, can be readily applied to extended networks [11].

In an extended network, because of the random Tx-Rx pairing, the distance between a transmitter and its receiver can grow with the network size. Based on an upper bound on the transport capacity (which equals the sum of products of rate and distance) in [7], the per-node throughput of an extended network is shown to also be upper bounded by  $1/\sqrt{n}$  [11].

### 3.2. INTERFERENCE-LIMITED NETWORKS WITH POISSON NODES

When nodes in the network are distributed according to a Poisson point process, the per-node throughput upper-bound of  $1/\sqrt{n}$  can be achieved. In [6], the authors establish a new routing scheme based on percolation theory that outperforms the nearest-neighbor forwarding scheme. Furthermore, this scheme assumes no exclusion regions around the receivers. Specifically, assume that nodes are distributed according to a Poisson point process of unit intensity on the square  $[0, \sqrt{n}] \times [0, \sqrt{n}]$  (an extended network). Source-destination pairs are picked uniformly. Then the per-user rate satisfies

$$R(n) \geq \frac{c_0}{\sqrt{n}} \quad (7)$$

where  $c_0$  is a constant.

The new routing scheme uses backbone routes (highways) and a 4-phase protocol. Divide the network into square grids of size of order  $\sqrt{n}/\log(\sqrt{n})$ . A backbone route is a path that connects from one side of the network to the other, using short, constant-length hops of a constant rate. With high probability, there exists at least one unique such backbone route within a slab of constant width (of order  $\log(\sqrt{n})$ ). There are  $\sqrt{n}$  such slabs, each containing a number of nodes of order  $\sqrt{n}$ . Furthermore, there is always a backbone path within a distance of order  $\log(\sqrt{n})$  from any node in the network. The 4-phase routing protocol then works as follows. In phase 1, drain all traffic to the backbone routes by direct (single hop) transmission. In phase 2, using multi-hop transmission, transport packets horizontally on the backbone routes. In phase 3, transport on the vertical backbones. In phase 4, deliver the packets to destination using direct transmission.

Each backbone route can carry traffic of order  $\sqrt{n}$  and is in fact the bottleneck in this protocol (the single-hop transmissions can have a per-node rate higher than  $1/\sqrt{n}$ ). Hence the achievable per-node rate of the network is of order  $1/\sqrt{n}$ .

### 3.3. FULLY COOPERATIVE NETWORKS WITH UNIFORM NODES

In contrast to the above interference-limited processing, which produces decreasing per-node throughput of order at most  $1/\sqrt{n}$ , when the physical-layer processing allows cooperation among the nodes, the per-node throughput can stay constant. In [15], the authors devise a multi-phase hierarchical scheme that achieves this linear sum-rate scaling law.

The new scheme operates based on hierarchical node clustering with ad hoc communication intra-clusters and MIMO communication inter-clusters. The nodes in the network are divided into clusters, each contains a number of nodes with the same order. Communication occurs in 3 phases. In the first phase, all nodes in the same cluster exchange information, ensuring every nodes in that cluster has all other nodes' messages. These intra-cluster communications work in parallel according to a 9-TDMA scheme, in which clusters sufficiently far apart operate in the same time slot to reduce interference among them. In the second phase, the clusters take turns to communicate with each other in a MIMO fashion. All (or half of) the nodes in each cluster now cooperate to send and receive information that belongs to one node inside that cluster, taking turns for all nodes. Because of this MIMO joint encoding and decoding, the inter-cluster communication can achieve a linear growth rate. In the third phase, nodes in each cluster again disseminate the received information among themselves, ensuring the information reaches its intended receiver.

This 3-phase communication protocol can operate in a hierarchical fashion. Each intra-cluster communication phase can again contain another 3-phase protocol, operating on smaller-size clusters. The lowest hierarchical level just uses the simple 9-TDMA scheme to exchange information within small clusters. The MIMO transmission then helps to disseminate information within a larger cluster. At the top level, the MIMO transmission will be on a global (network) scale. It is then shown that each higher level can achieve a better throughput than the previous one. Intuitively, the linear growth rate in the MIMO phases can outweigh the overheads in the information exchanging phases, resulting in an overall growth rate approaching linear.

For dense networks, applying to channels with path loss  $\alpha > 2$  and random phases, this hierarchical protocol achieves a per-node throughput which is asymptotically constant. In other words, with high probability, the sum rate grows linearly as

$$C(n) \geq K_\epsilon n^{1-\epsilon} \quad (8)$$

for any  $\epsilon > 0$ , where  $K_\epsilon > 0$  is a constant independent of  $n$ .

For extended networks, it is shown that this hierarchical scheme is optimal for path loss  $2 < \alpha \leq 3$  and achieves a sum rate scaling as

$$C(n) \sim n^{2-\alpha/2} \quad (9)$$

In this region, the scaling rate is higher for lower path loss. For  $\alpha > 3$ , the nearest-neighbor multihop scheme is optimal and produces a sum rate scaling independent of the path loss as

$$C(n) \sim \sqrt{n}. \quad (10)$$

Note that for extended networks, the sum rate scaling is sub-linear.

#### 4. Cognitive Networks

In this section, we consider a cognitive network with two types of users: primary and secondary. As an initial study, we formulate a simple network with only one primary user and  $n$  secondary (cognitive) users [16]. Such a network can represent a broadcasting scenario, for example, in the TV or cellular networks, in which the cognitive devices access the spectrum without harming the primary user.

##### 4.1. NETWORK MODEL

The considered network is an extended network with a slightly different geometry from the previous models. Instead of the square shape, we define a circular network, in which the primary transmitter is at the center, and the primary receiver is within a certain radius  $R_0$  (the precise location of the primary receiver may be unknown to the cognitive users). Outside this radius is a cognitive band, which contains the  $n$  cognitive users distributed *randomly and uniformly*, as illustrated in Figure 1.

We consider the scenario in which the cognitive transmitters are uniformly distributed such that their density is a constant  $\lambda$ . The network outer radius  $R$  therefore grows as the number of cognitive users  $n$  increases. We assume that each cognitive transmitter communicates with a receiver within a *bounded distance*  $D_{\max}$ , independent of the network size. The cognitive communication therefore occurs in a *single hop*. This assumption appears reasonable for secondary spectrum usage, which is opportunistic in nature and hence is often a local, single-hop transmission. Furthermore, we assume that any interfering transmitter must be at a non-zero distance  $\epsilon$  ( $\epsilon > 0$ ) away from the interfered receiver, again a practically reasonable assumption.

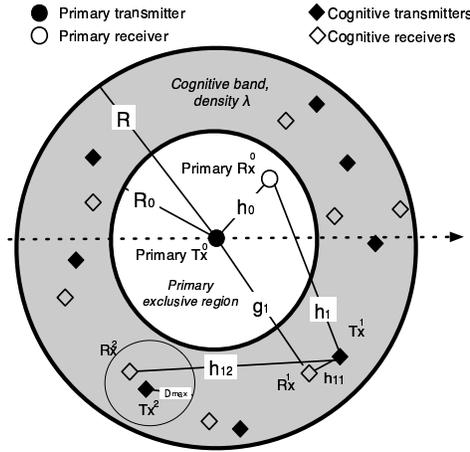


Figure 1. A cognitive network: a single primary transmitter  $Tx^0$  is placed at the origin and wishes to transmit to its primary receiver  $Rx^0$  in the circle of radius  $R_0$  (the *primary exclusive region*). The  $n$  cognitive nodes are randomly placed with uniform density  $\lambda$  in the shaded *cognitive band*. The cognitive transmitter  $Tx^i$  wishes to transmit to a single cognitive receiver  $Rx^i$  which lies within a distance  $< D_{max}$  away. The cognitive transmissions must satisfy a primary outage constraint.

We are interested in two main questions: the scaling law of the cognitive users, and the radius  $R_0$  which ensures a given outage constraint on the primary user. This outage defines the probability that the received signal (or rate) of the primary user is below a certain level (due to noise and interference). The radius  $R_0$  then specifies a *primary exclusive region*, inside of which no cognitive users may operate (either transmit or receive). Outside this region, however, the cognitive users may freely communicate among themselves.

#### 4.2. LINEAR SCALING OF THE COGNITIVE NETWORK THROUGHPUT

We consider a channel with path loss  $\alpha > 2$ . Assume that the primary user transmits with power  $P_0$ , and each cognitive user transmits with power  $P$ . The network is *interference-limited* such that each receiver treats other users' signals as noise. It can then be shown [16] that the average sum rate of this cognitive network scales linearly with the number of users  $n$ . Specifically, with high probability, the per-user rate stays constant and satisfies

$$R(n) \geq \log \left( 1 + \frac{P_{\min}}{\sigma_{\max}^2 + I_{\infty}} \right), \tag{11}$$

where  $P_{\min} = P/D_{\max}^\alpha$ ,  $\sigma_{\max}^2 = \sigma_n^2 + P_0/R_0^\alpha$ , and  $\sigma_n^2$  is the thermal noise power. Here  $I_\infty$  is the worst-case interference and is given by

$$I_\infty = \frac{2\pi\lambda P}{(\alpha - 2)\epsilon^{\alpha-2}}. \quad (12)$$

From the above expression, we see that  $\epsilon > 0$  is critical in achieving the constant per-node throughput. This condition is equivalent to an exclusion region around each cognitive receiver, inside of which no other cognitive transmitters may operate. The radius  $\epsilon$  of this region can be arbitrarily small without affecting the scaling law, but it does affect the achievable rate itself.

Another critical parameter in achieving the linear sum throughput is the bounded maximum Tx-Rx distance  $D_{\max}$ . While  $D_{\max}$  can be as large as desired, it does not grow with the network size when  $n \rightarrow \infty$ . This is in contrast to the previous models of interference-limited ad hoc networks (Sections 3.1 and 3.2), in which the transmitters and receivers are *randomly* paired up. Hence in the worst case, the Tx-Rx distance in these ad hoc networks can grow as  $\sqrt{n}$ , leading to the  $1/\sqrt{n}$  per-node throughput scaling.

In other words, the cognitive network performs single hop transmission instead of multihop forwarding. This key difference enables the linear growth in the sum throughput even in the interference-limited regime. We note that single hop transmission is both practical and favorable in the opportunistic-based communication seen in a cognitive setting.

#### 4.3. THE PRIMARY EXCLUSIVE REGION

To bound the primary exclusive region, we consider the worst case when the primary receiver is at the edge of this region, on the circle of radius  $R_0$  [17]. The outage constraint must also hold in this (worst) case, such that

$$Pr [C_0 \leq T] \leq \beta \quad (13)$$

where  $C_0$  is the primary user's transmission rate,  $T$  is a given threshold and  $\beta$  is the specified outage level ( $0 < \beta < 1$ ). Outages occur here because of the random cognitive user placement, since we assume no fading in the channels (3).

By upper bounding the average interference to the primary receiver, we can bound the radius  $R_0$  of the primary exclusive region. Here we provide the results for a specific case, when the power path loss  $\alpha = 4$ . With an infinite number of cognitive users (the worst case), the average

interference from these cognitive users to the primary user is given by

$$E[I_0] = \lambda\pi P \frac{(R_0 + \epsilon)^2}{\epsilon^2(2R_0 + \epsilon)^2}. \quad (14)$$

Using this average interference in the outage constraint (13), we obtain a condition on  $R_0$  as

$$\frac{(R_0 + \epsilon)^2}{\epsilon^2(2R_0 + \epsilon)^2} \leq \frac{\beta}{\lambda\pi P} \left( \frac{P_0/R_0^4}{2^{C_0} - 1} - \sigma_n^2 \right). \quad (15)$$

Given the system parameters:  $P_0$  (the primary transmit power),  $C_0$  (the outage capacity),  $\beta$  (the outage probability),  $P$  (the cognitive transmitter power),  $\lambda$  (the cognitive user density), and  $\sigma_n^2$  (the noise power), (15) may be used to jointly design the exclusive region radius  $R_0$  and the gap  $\epsilon$  to meet the desired outage constraint.

## 5. Conclusion

In this paper, we provided an overview on the scaling laws of ad hoc and cognitive networks. Specifically, for classical ad hoc networks, results on how the sum-rate scales as the number of nodes  $n$  tends to infinity was presented for three different sets of network assumptions. In a random network with uniformly distributed nodes, multi-hop nearest-neighbor routing achieves a per node throughput scaling of the order  $1/\sqrt{n \log(n)}$ . In a random network with nodes distributed according to a Poisson point process, a routing scheme based on information “highways” achieves a per-node throughput scaling of the order  $1/\sqrt{n}$ . These results apply to interference-limited networks. In a network that nodes may cooperate, on the other hand, a hierarchical cooperation strategy can achieve a constant throughput per node. We next explored the sum-rate scaling of a cognitive network which operates in secondary licensed spectra. The secondary, or cognitive, nodes must transmit while satisfying an outage constraint for a single primary user located at the center of the network. We showed that when the cognitive transmitter-receiver pairs are within a bounded distance, then by using *single-hop transmission*, a constant per-node throughput may be achieved by the cognitive network without affecting the primary user (according to the outage specification). Future directions for cognitive networks include considering multiple primary nodes, fading, as well as different cognitive capabilities and transmission schemes for the secondary nodes.

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