Optimum Space-Time Transmission for a High K Factor Wireless Channel with Partial Channel Knowledge

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***Abstract*— We study the optimum transmission scheme that maximizes ergodic capacity of a 2×1 multiple-input single-output (MISO) system, when the channel knowledge at the transmitter is characterized by a known gain ratio and a known probability density function of the phase shift between antennas. Such a channel scenario can arise in a forward link at the base station when there is a single direct path propagation. We show that the optimum transmit solution is beamforming on the mean value of the phase shift with unequal power input to the antennas. When the phase is completely unknown, the solution reduces to a single antenna transmission.**

I. Introduction

Recent works in multiple-input multiple-output (MIMO) wireless channel capacity and coding have shown that channel knowledge at the transmitter, in either full or partial forms, can increase the channel capacity and performance considerably [6]-[12]. While zero-mean channels have been largely the focus of the existing work, it is also often found in practice that the wireless channel has a *non-zero mean* [2], [3]. In other words, a finite K factor, which is the ratio of the power in the fixed component to that of the variable component in the channel, exists. This motivates the study of transmit schemes for K-factor channels.

In this paper, we study a limiting case when the K factor is infinity, which corresponds to a direct dominant path propagation. The results however could be applicable to channels with high K factors, say 20dB, which occur in practice [3]. The receiver is assumed to know the channel perfectly. The transmit channel knowledge model assumes a perfectly known antenna gain ratio, including equal-gain, and a random phase shift with a known probability density function (PDF). This scenario is typical in the forward link at a base station with direct path propagation and large spacing ($\sim$ 10 carrier wavelengths) between the two transmit antennas. This model of channel knowledge differs from models considered in the aforementioned work, where long-term first and second order statistic information of the channel are usually assumed, together with complex Gaussian distributed channel coefficients. The channel coefficients in this model represent a non-fading wireless link which has a phase shift uncertainty.

We derive the optimum transmission scheme from the ergodic capacity point of view, based on the known channel gain ratio, the PDF of the channel phase shift, and the SNR (Signal to Noise Ratio). The optimum scheme can be divided into two categories depending on the channel gain ratio factor. If the channel gain is unequal (i.e. the ratio is different from 1), the optimum scheme is to always do beamforming on the mean of the channel phase shift, with power at each antennas adjusted according to the channel gain ratio. When the two antennas have equal gain, then the optimum scheme includes, but is not limited to, beamforming. For this case, optimum signals from two transmit antennas can also be designed according to a given full-rank covariance matrix. In both cases however, when there is no phase knowledge between the two antennas at the transmitter, the optimum scheme reduces to single antenna transmission.

In the next section, we give details about the channel model and assumptions. Section 3 sets up the problem based on ergodic capacity and summarizes the main results. The optimum signal phase shift is then established in Section 4. Section 5 presents results for the optimum signal power allocation and covariance magnitude for both cases of unequal and equal channel gains. Section 6 gives some simulation examples of the results being applied specifically to Ricean phase distribution [14], [17]. We close with some concluding remarks in Section 7.

Notations used in this paper: E is expectation, $(.)^*$ is complex conjugate and $(.)^\star$ is the optimum value.

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II. Channel Model

Consider a MISO system with two transmit antennas. Assuming a single direct propagation path and narrowband antenna array [2], the channel can be depicted as in Figure 1. The propagation paths from two antennas differ by a channel gain ratio α and a phase shift ϕ

$$h_2(t) = \alpha e^{j\phi} h_1(t) \ ,$$

where $h_1(t)$ and $h_2(t)$ are the channels seen from the first and second antennas, respectively. Assuming high K factor, we can suppress the time dependence of the channel amplitudes and write the channel simply as a row vector $[h_1 \ \ h_2]$.

Let d be the distance between the two antennas, θ be the angle of departure and λ_c be the carrier wavelength, the phase shift between two antennas is given by

$$\phi = 2\pi \frac{d}{\lambda_c} \sin\theta \ .$$

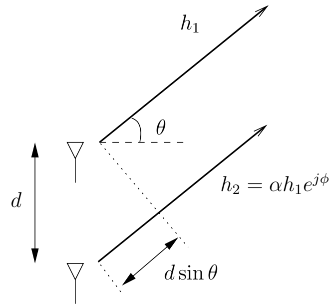


Fig. 1. Single path propagation model.

Due to the large distance (~ 10 wavelengths) between the two antennas at high carrier frequencies, the phase shift between two antennas is highly variable in response to a small change in the angle of departure. For example, if the angle of departure θ is assumed to be uniformly distributed within $[-\pi/3, \pi/3]$ at the basestations, together with Doppler spread on f and possible changes or jitters of carrier frequencies, the distribution of ϕ will look almost uniform between $[-\pi, \pi]$.

The difference in antenna gains (when $\alpha \neq 1$) is caused by the local scattering from mounting structure (walls, rooftops ...) near the antennas and is often found in practice. While this gain ratio α also exhibits a dependence on the angle of departure, which is a function of the antenna pattern, its sensitivity to the departure angle is much less than that of the phase shift. We therefore assume that this gain ratio can be tracked and measured accurately.

In this study, we assume that the receiver has perfect knowledge of the channel, while the transmitter has some partial channel knowledge. The transmitter can obtain the channel in the forward path by estimating the reverse channel in TDD (Time Division Duplexing) systems, or through feedback from the receiver in FDD (Frequency Division Duplexing) systems. In both cases, there is likely to be an error in the estimation due to the time offset or lag between the channel measurement and its use, hence the transmitter can only obtain partial channel knowledge. The channel can be characterized by antenna gains and a phase shift between the two antennas. Due to high K factor, the antenna gains are likely to be very stable and can be estimated accurately. We therefore assume perfectly known α and h_1. The antenna phase shift, however, is highly variable due to the large separation between antennas, leading to errors in the phase estimate. We assume the PDF of the phase shift ϕ is known but not the exact value of the phase. This phase shift distribution is circular between $-\pi$ and π. The precise shape of the distribution depends on the channel characteristics and the measurement method. A Dirac delta distribution function corresponds to exact phase knowledge, whereas a uniform distribution means no phase information. In fast time varying channels, the phase measurements are more error prone, hence the distribution will tend toward uniform.

Our analysis requires that the phase shift distribution is *symmetric* around the mean ϕ_0. The exact power allocation at the two transmit antennas does depend on the actual PDF of the phase shift. We first carry out the analysis for general phase PDFs, and then apply the analysis results to Ricean phase distribution specifically for numerical simulations. The Ricean phase distribution can be parameterized to cover the range from uniform phase distribution (i.e. unknown phase shift) to delta function distribution (i.e. exact phase knowledge) and will be discussed in more details in the numerical result section.

III. Problem Overview

A. Problem Setup

We use the ergodic channel capacity under the sum power constraint on transmit antennas as the optimization criterion. Since the receiver has full knowledge of the channel $[h_1 \;\; h_2]$, the ergodic capacity of the channel is achieved by Gaussian input signal $[x_1 \;\; x_2]^T$ with zero mean [1] and a *covariance matrix* $\mathbf{R}_{xx}$ which satisfies

$$\begin{aligned} \max_{\mathbf{R}_{xx}} \quad & E \log(1 + \gamma \mathbf{h} \mathbf{R}_{xx} \mathbf{h}^*) \\ \text{s.t.} \quad & \text{tr}(\mathbf{R}_{xx}) = 1 \; , \end{aligned} \tag{1}$$

where γ is the total signal to noise ratio with appropriate normalization.

We can absorb h_1 into γ and write the effective channel as

$$\mathbf{h} = [1 \quad \alpha e^{j\phi}] \; ,$$

where $0 \leq \alpha \leq 1$. Taking into account the total transmit power constraint $\text{tr}(\mathbf{R}_{xx}) = 1$, the transmit signal covariance matrix $\mathbf{R}_{xx}$ can be expressed as

$$\mathbf{R}_{xx} = \begin{bmatrix} \eta & \frac{1}{2}\rho e^{-j\psi} \\ \frac{1}{2}\rho e^{j\psi} & 1-\eta \end{bmatrix} . \tag{2}$$

Here η is the *fraction of total power* allocated to the first antenna, ψ is the *signal phase shift* and ρ is *twice the magnitude of the covariance* between signals transmitted from the two antennas defined as

$$\rho = 2\big|E[x_1 x_2^*] - E[x_1]E[x_2^*]\big| = 2\big|E[x_1 x_2^*]\big| \; ,$$

where the last equality results from the signals having zero-mean. In the followings we will call ρ simply as the covariance magnitude.

The three variables η, ψ and ρ define the transmission scheme. The constraints on these variables become

$$\begin{aligned} 0 \;\; &\leq \;\; \eta \;\; \leq \;\; 1 \\ -\pi \;\; &\leq \;\; \psi \;\; \leq \;\; \pi \\ 0 \;\; &\leq \;\; \rho \;\; \leq \;\; 2\sqrt{\eta(1-\eta)} \; . \end{aligned} \tag{3}$$

The bounds on η follow immediately from its definition, whereas the bounds on ψ result from a predefined domain of the signal phase shift. The upper bound on ρ comes from the covariance relation

$$\big|E[x_1 x_2^*]\big| \leq \sqrt{E|x_1|^2 E|x_2|^2} \tag{4}$$

for zero-mean random variables. This bound can also be obtained via the positive semidefinite property of the covariance matrix $\mathbf{R}_{xx}$.

With these channel and signal models, and the assumption that the phase distribution is symmetric around its mean ϕ_0, the average mutual information can be written as

$$\mathcal{I} = E\log(1+\gamma\mathbf{h}\mathbf{R}_{xx}\mathbf{h}^*) \quad (5)$$
$$= \int_{-\pi}^{\pi}\log\Big[\gamma(1-\alpha^2)\eta+\gamma\alpha\rho\cos(\phi+\psi_0)+\gamma\alpha^2+1\Big]f(\phi)d\phi$$

where $\psi_0 = \phi_0+\psi$. Note that $f(\phi)$ is the phase PDF function translated to be centered at its mean, that is $f(\phi)$ is symmetric around zero. In the following sections, we will use this translated phase PDF $f(\phi)$ in all derivations. We are interested in maximizing (5) by choosing η, ψ and ρ subject to the constraints (3).

B. Summary of Results

The optimum transmission scheme is defined by the transmit covariance matrix $\mathbf{R}_{xx}$, which in turn is defined by η, ψ and ρ. These are found based on the known channel parameters at the transmitter, which are the channel phase shift distribution $f(\phi)$, the channel gain ratio α and the SNR γ. The main results can be summarized as

- The optimum signal phase shift $\psi^\star$ is the negative of the estimate channel phase shift ϕ_0 or that plus π, depending on the specific phase shift distribution $f(\phi)$. The $\psi^\star$ value is independent of α and γ. This is derived in Section IV.
- When the channel gain is unequal ($\alpha < 1$), the optimum transmission scheme is always beamforming with unequal power allocation to the two transmit antennas. The optimum $\rho^\star$ is a function of $\eta^\star$, and $\eta^\star$ is a function of the phase shift distribution $f(\phi)$, the channel gain ratio α and the SNR γ. This is derived in Section V-A.
- When the channel gain is equal ($\alpha = 1$), the optimum solution can either be beamforming or space-time coding with correlated signals. In this case, η vanishes in the average mutual information expression (5). $\rho^\star$ is a function of $f(\phi)$ and γ, then $\eta^\star$ can be chosen arbitrarily within its range subject to the inequality on $\rho^\star$ in (3). This is analyzed in Section V-B.

IV. Optimum Signal Phase Shift $\psi^\star$

The optimum signal phase shift $\psi^\star$ is *independent* of the channel gain ratio α and the SNR γ, and hence, is treated separately in this section.

Theorem 1: The optimum phase shift $\psi^\star$ between the transmit signals from two antennas is the negative of the estimated channel phase shift ϕ_0 or that plus π, depending on the channel phase shift distribution $f(\phi)$. That is

$$\psi^\star = -\phi_0 \quad \text{or} \quad \psi^\star = \pi-\phi_0 \ .$$

Proof. The original problem (1) is a convex optimization problem and hence has a unique solution, which leads to a unique solution of $\psi^\star$. Due to symmetry of the phase distribution, from (5), we can rewrite the average mutual information as

$$\mathcal{I} = \int_0^{\pi}\log\big[\,p^2+2p\gamma\alpha\rho\cos\psi_0\cos\phi+ \gamma^2\alpha^2\rho^2(\cos^2\psi_0+\cos^2\phi-1)\big]f(\phi)d\phi\ ,$$

where $p = \gamma(1-\alpha^2)\eta+\gamma\alpha^2+1$ and $\psi_0 = \phi_0+\psi$. The mutual information expression can be rewritten as a function of $z = \cos\psi_0 = \cos(\phi_0+\psi)$. The optimization can be carried out with respect to z instead of ψ, then the optimum value $\psi^\star$ can be derived from $z^\star$. If the optimum $z^\star$ is not 1 or -1, then there will be two different values of the optimum phase shift $\psi^\star$ within the range $[-\pi,\pi]$ that satisfy $\cos(\phi^\star+\phi_0) = z^\star$, as cosine is an even function. This in turn violates the uniqueness of $\psi^\star$. Therefore the optimum value of z must be either 1 or -1, which leads to the optimum signal phase shift $\psi^\star = -\phi_0$ or $\psi^\star = \pi-\phi_0$ respectively. This is the result of the symmetry of the channel phase shift distribution. □

The specific value for $\psi^\star$ depends on the phase shift distribution function $f(\phi)$. The choice can be made by examining the mutual information $\mathcal{I}$ at the two boundary values $\psi = -\phi_0$ and $\psi = \pi-\phi_0$, then choose the value which makes $\mathcal{I}$ larger. With $\psi = -\phi_0$, the average mutual information (5) becomes

$$\mathcal{I}|_{\psi=-\phi_0} = 2\int_0^{\pi}\log\big(p+\gamma\alpha\rho\cos\phi\big)f(\phi)d\phi\ .$$

A similar expression can be obtained for the case $\psi = \pi-\phi_0$. The difference between average mutual information at the two candidate values for $\phi^\star$ is

$$\begin{aligned}\Delta\mathcal{I} &= \mathcal{I}|_{\psi=-\phi_0}-\mathcal{I}|_{\psi=\pi-\phi_0}\\ &= 2\int_0^{\pi}\log\Big(\frac{p+\gamma\alpha\rho\cos\phi}{p-\gamma\alpha\rho\cos\phi}\Big)f(\phi)d\phi\ .\end{aligned}$$

The logarithmic expression is anti-symmetric around $\pi/2$, therefore the above difference can be rewritten in the form

$$\Delta\mathcal{I} = 2\int_0^{\pi/2}\log\Big(\frac{2p}{p-\gamma\alpha\rho\cos\phi}-1\Big)\Big[f(\phi)-f\Big(\phi+\frac{\pi}{2}\Big)\Big]d\phi\ .$$

Since the logarithmic expression in the above integral is non-negative within the integral range, we can obtain the sign of $\Delta\mathcal{I}$ simply without having to evaluate the integral explicitly if $f(\phi)-f\big(\phi+\frac{\pi}{2}\big)$ does not change sign within $0\le\phi\le\pi/2$. More specifically, if the translated phase distribution function $f(\phi)$ is monotonous within the range $0\le\phi\le\pi$, then the choice amongst $\phi = 0$ and $\phi = \pi$, which makes $f(\phi)$ larger, corresponds to the optimum $\psi^\star$ value being $-\phi_0$ or $\pi-\phi_0$ respectively. This monotonicity applies to the Ricean phase distribution which we use in numerical simulations later. Since $f(\phi)$ is circular within $-\pi\le\phi\le\pi$, we can always rotate the distribution so that $f(0) > f(\pi)$. Therefore without loss of generality, we assume that the channel phase shift distribution is such that the optimum signal phase shift is $\psi^\star = -\phi_0$ in the next section.

V. Optimum Signal Power $\eta^\star$ and Covariance Magnitude $\rho^\star$

In this section we will derive the optimum set of η and ρ. It turns out that the cases of unequal channel gain ($\alpha < 1$) and equal channel gain ($\alpha = 1$) have significantly different impact on the optimum $\eta^\star$ and $\rho^\star$. While the solution of unequal channel gain case can be applied to the equal case, the latter has a larger solution space. Therefore we analyze these two cases separately.

A. Unequal channel gains

We assume without loss of generality that the first antenna always has a higher gain than the second antenna, thus α is strictly less than 1. The optimization problem (1) now becomes

$$\begin{aligned} \max \ & \int_0^{\pi} \log\big[\gamma(1-\alpha^2)\eta + \alpha\rho\gamma\cos\phi + \gamma\alpha^2 + 1\big] f(\phi) d\phi \\ \text{s.t.} \ & 0 \ \leq \ \eta \ \leq \ 1 \\ & 0 \ \leq \ \rho \ \leq \ 2\sqrt{\eta(1-\eta)} \ . \end{aligned} \tag{6}$$

Optimum signal covariance magnitude $\rho^\star$

Theorem 2: With $\alpha < 1$, the optimum magnitude of the covariance between the transmit signals from the two antennas is

$$\rho^\star = 2\sqrt{\eta(1-\eta)} \ . \tag{7}$$

Hence the transmit signals has the form

$$x_2 = \zeta e^{-j\phi_0} x_1 \ , \tag{8}$$

with ζ given by

$$\zeta = \sqrt{\frac{1-\eta}{\eta}} \ .$$

In other word, the optimum transmission scheme reduces to simple beamforming with unequal power at each antenna.

Proof. Problem (6) is a convex optimization problem. Form the Lagrangian functional

$$\begin{aligned} \mathcal{L}(\eta, \rho, \lambda) \ = \ & E\log\big[\gamma(1-\alpha^2)\eta + \alpha\rho\gamma\cos\phi + \gamma\alpha^2 + 1\big] \\ & -\lambda\big[\rho - 2\sqrt{\eta(1-\eta)}\big] \ , \end{aligned}$$

where $\lambda \geq 0$ is the Lagrange multiplier. Then the optimizers $\eta^\star$ and $\rho^\star$ are the solutions of the equations formed by setting the partial derivatives of $\mathcal{L}(\eta, \rho, \lambda)$ to zero. In particular, setting the partial derivative with respect to η to zero leads to

$$E\Big[\frac{\gamma(1-\alpha^2)}{\gamma(1-\alpha^2)\eta + \alpha\rho\gamma\cos\phi + 1 + \gamma\alpha^2}\Big] = \lambda\frac{2\eta - 1}{\sqrt{\eta(1-\eta)}} \ .$$

For $\alpha < 1$, the left-hand-side of the above expression is strictly greater than 0 for all distributions of ϕ as the expression under the expectation is always positive. Thus $\lambda^\star > 0$ and $\eta^\star > \frac{1}{2}$. Since $\lambda^\star$ is strictly positive, it means that the upper constraint on ρ is tight (Karush-Kuhn-Tucker conditions [4]), hence $\rho^\star = 2\sqrt{\eta(1-\eta)}$. Another way to arrive at this result is by equating the partial derivative of $\mathcal{L}(\eta, \rho, \lambda)$ with respect to ρ to zero to get

$$\frac{\partial \mathcal{I}}{\partial \rho} = E\Big[\frac{\alpha\gamma\cos\phi}{\gamma(1-\alpha^2)\eta + \alpha\rho\gamma\cos\phi + 1 + \gamma\alpha^2}\Big] = \lambda > 0 \ .$$

Therefore the average mutual information $\mathcal{I}$ is increasing in ρ at the optimum point. This means the optimum $\rho^\star$ is achieved at its maximum value.

This maximum covariance magnitude can be achieved only when the signal sent from one antenna is a scaled version of the signal sent from the other antenna, following the equality condition on (4). Applying the phase shift result of Theorem 1, the transmit signals become $x_2 = \zeta e^{-j\phi_0} x_1$. The covariance magnitude becomes $\rho = 2E[x_1 x_2^*] = 2\zeta\eta = 2\sqrt{\eta(1-\eta)}$, which leads to the value of ζ as given in the Theorem. □

Hence the optimum transmit strategy is to do *beamforming* all the time, with the power at each antenna adjusted according to the channel parameters. The optimum covariance matrix $\mathbf{R}_{xx}$ always has rank one in this case.

Optimum power allocation $\eta^\star$

Replacing the optimum $\rho^\star$ into the average mutual information in (6), the problem then becomes finding $\eta \in [0, \ 1]$ to maximize the following expression

$$E\log\big[(1-\alpha^2)\eta + 2\alpha\sqrt{\eta(1-\eta)}\cos\phi + \alpha^2 + \frac{1}{\gamma}\big] \ .$$

Since the above expression is concave in η, the optimum $\eta^\star$ is the solution of

$$E\Big[\frac{1-\alpha^2 + \frac{1-2\eta}{\sqrt{\eta(1-\eta)}}\alpha\cos\phi}{(1-\alpha^2)\eta + 2\alpha\sqrt{\eta(1-\eta)}\cos\phi + \alpha^2 + \frac{1}{\gamma}}\Big] = 0 \ . \tag{9}$$

The optimum $\eta^\star$ is a function of $f(\phi)$, α, γ.

B. Equal channel gains

In this section we treat the case $\alpha = 1$. With the optimum signal phase $\psi^\star = -\phi_0$, the average mutual information becomes

$$\mathcal{I} = 2\int_0^{\pi} \log\big(\rho\gamma\cos\phi + \gamma + 1\big) f(\phi) d\phi \ .$$

Notice that the signal power allocation η does not appear in this expression as a result of the equal channel gain. Therefore in this case, the covariance magnitude ρ can be found independently of η and the maximization can be taken over $0 \leq \rho \leq 1$.

Optimum signal covariance magnitude $\rho^\star$

Since the above expression is concave in ρ, the optimum $\rho^\star$ is the solution of

$$\frac{\partial \mathcal{I}}{\partial \rho} = 2\int_0^{\pi} \frac{\gamma\cos\phi}{\rho\gamma\cos\phi + \gamma + 1} f(\phi) d\phi = 0 \ . \tag{10}$$

The optimum $\rho^\star$ depends on the specific phase distribution $f(\phi)$ and the SNR γ.

Optimum power allocation $\eta^\star$

Here the optimum $\rho^\star$ and η are only related to each other through the inequality

$$\rho^\star \leq 2\sqrt{\eta(1-\eta)} \ . \tag{11}$$

Hence one can choose any power allocation value η that satisfies this relation and design the signal according to the obtained optimum $\mathbf{R}_{xx}$. The rank of this covariance matrix is not restricted to be one as in the unequal channel gain case. The choice of $\eta^\star$, which influences the rank of $\mathbf{R}_{xx}$, therefore can be divided into two general categories:

- $\mathbf{R}_{xx}$ *rank one - Beamforming:* Here we pick the value of $\eta^\star$ that meets the bound (11) with equality, which gives

$$\eta^\star = \frac{1}{2}(1 \pm \sqrt{1-(\rho^\star)^2}) \ . \tag{12}$$

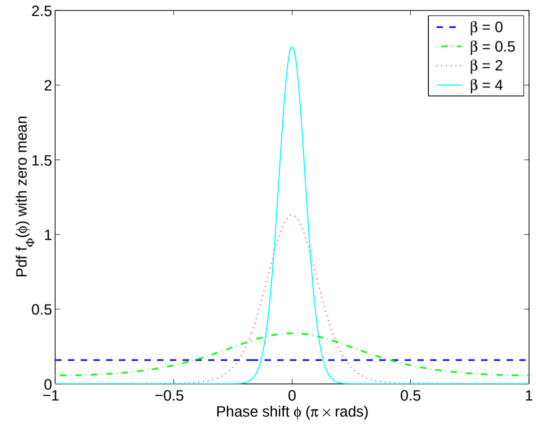


Fig. 2. Ricean phase distribution

This is the same solution as the optimum scheme for unequal channel gain case (7). The optimum signal design is then $x_2 = \zeta e^{-j\phi_0} x_1$, where ζ is given by

$$\zeta = \frac{\rho}{1 \pm \sqrt{1-\rho^2}} \,.$$

- $\mathbf{R}_{xx}$ *full rank:* This can be done by picking a value of η that satisfies the inequality (11) strictly. The signal design problem then becomes finding a coding scheme for the given covariance matrix $\mathbf{R}_{xx}$. A specific choice is $\eta = \frac{1}{2}$, which makes the signals from the two antennas have equal power. The optimum signal then has to be designed such that x_1 and x_2 have identical Gaussian distributions with a correlation factor equal to $\rho^\star/2$.

As a special case for both of the above categories, when there is *no phase estimate* (equivalent to uniform phase shift distribution), then the optimum solution is $\rho^\star = 0$, which means sending independent zero-mean Gaussian signals from two antennas with the only constraint being that their powers add up to one. Using a single antenna and putting all the transmit power there also achieves the capacity with no randomness, hence single antenna transmission is preferred in this case. That is, $\eta^\star = 1$.

VI. Simulation Examples

We use Ricean phase distribution for the channel phase shift in the simulations. This distribution arises from the phase of a constant phasor plus random zero-mean complex Gaussian noise with equal variance on the real and imaginary parts [14], [17]. The *phase estimate quality* can be conveniently described by the Ricean factor β. Assuming an estimated mean ϕ_0 with a given estimate quality β, and denoting $\tilde{\phi} = \phi - \phi_0$, the phase shift distribution function is

$$f_{\mathbf{\Phi}}(\phi) = \frac{e^{-\beta^2}}{2\pi}\left\{1 + \sqrt{\pi}\beta\cos\tilde{\phi}\, e^{\beta^2\cos^2\tilde{\phi}}\left[1 + \mathrm{erf}(\beta\cos\tilde{\phi})\right]\right\}. \quad (13)$$

If $\beta = 0$, the phase distribution is uniform, corresponding to no phase estimate. When $\beta \to \infty$, the distribution converges to the Dirac delta function, which means that the estimate is exact. A plot of the phase distribution with estimated mean $\phi_0 = 0$ at various values of β is given in Figure 2.

A. Unequal channel gains

We solve equation (9) numerically to find $\eta^\star$. It turns out that the SNR γ has a very little effect on $\eta^\star$, which can be seen from this equation as $1/\gamma$ can be ignored for reasonably large values of γ. Simulation results show that we get practically the same value of $\eta^\star$ for all $\gamma \geq -20$dB. Figure 3 shows the plot of the optimum power allocation $\eta^\star$ as a function of the channel gain ratio α and the phase estimate quality β, at SNR $\gamma = 10$dB.

When $\eta = 1$, it means that only one antenna is used. This is the case when no phase estimate exists ($\beta = 0$). In such situations, using only the stronger antenna to transmit is optimum regardless of the actual α value ($\alpha < 1$ here). As the phase estimate quality increases, the power distributes to both antennas unequally. The scheme approaches transmit maximum ratio combining (MRC) beamforming, which is optimum when the channel is known perfectly at the transmitter. The MRC beamforming power allocation is a function of α and is given as

$$\eta_{\text{MRC}} = \frac{1}{1+\alpha^2} \,. \quad (14)$$

Figure 4 shows slices of the optimum power allocation versus the phase estimate quality and channel gain ratio. In the power versus phase estimate quality plot, the power allocation η levels off for $\beta \geq 3$ approximately at all channel gain ratios. These levels are the MRC power allocations at the corresponding channel gain ratio α. The same effect is reflected in the power versus channel gain ratio plot. The power allocation plots for $\beta \geq 3$ are almost indistinguishable, and correspond to a plot of the MRC power allocation (14) versus α. Thus practically, MRC beamforming can be close to optimum even at imperfect phase estimates.

B. Equal channel gains

Solving equation (10) with the Ricean phase distribution by numerical means, we obtain the plot for the optimum $\rho^\star$ in Figure 5. The value of $\rho = 1$ means beamforming where signal sent from one antenna is a scaled version of signal sent from the other, whereas $\rho = 0$ means independent signals from the two antennas.

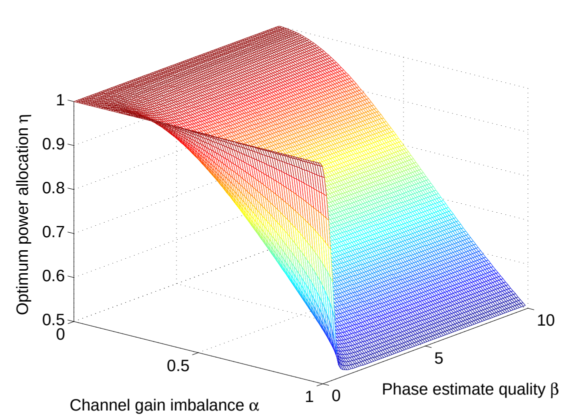
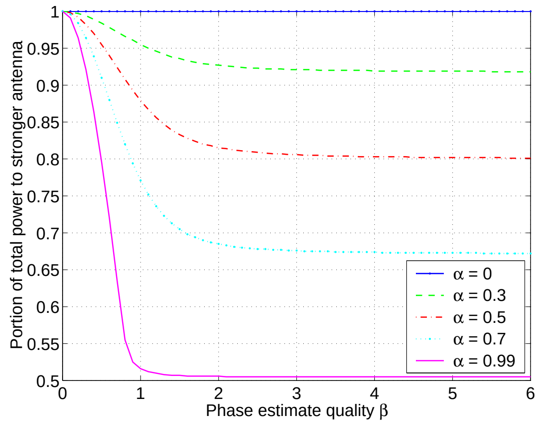
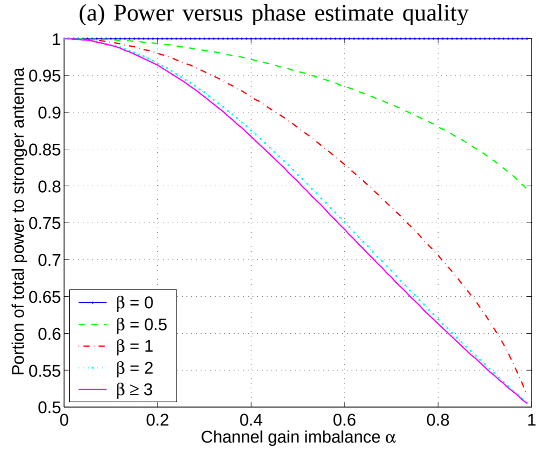


Fig. 3. Optimum $\eta^\star$ in unequal channel gain case at SNR=10dB.



(a) Power versus phase estimate quality



(b) Power versus channel gain ratio

Fig. 4. Optimum power allocation for unequal channel gain.

In case of a beamforming solution ($\mathbf{R}_{xx}$ rank one), the power split between the two antennas (12) is regulated according to the phase estimate quality β and the SNR γ. A plot of the optimum power allocation $\eta^\star$ versus the phase estimate quality β at various SNRs is shown in Figure 6(a). Since the roles of the two antennas here are symmetric, we only show values such that $\eta^\star \geq 0.5$. The lines in this plot correspond to the edge of the surface in Figure 3 at $\alpha = 1$ at different SNRs. Notice that at this edge, $\eta^\star$ depends quite significantly on the SNR.

If the phase estimate quality β is *above* a certain threshold, which is a function of the SNR γ, then the integral in (10) is always non-negative for $0 \leq \rho \leq 1$, which leads to *only beamforming* being optimum. In this particular case, the beamforming corresponds to $\rho^\star = 1$ and $\eta^\star = \frac{1}{2}$, that is, both antennas transmit with equal power. This threshold is plotted in Figure 6(b).

C. Capacity comparison with and without channel knowledge

In this section we provide a quantitative analysis of the gain in capacity obtained by the partial channel knowledge at the transmitter. For both cases of unequal and equal channel gain, the capacity with no phase estimate available is

$$\mathcal{C}_0 = \log(1+\gamma) \ ,$$

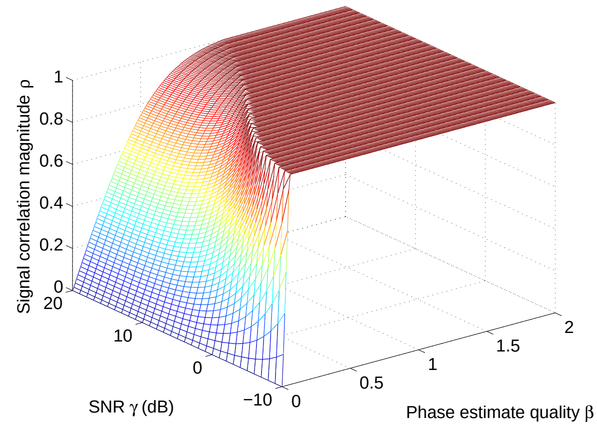


Fig. 5. Optimum $\rho^\star$ in equal channel gain case.

and the capacity with perfect phase estimate is

$$\mathcal{C}_1 = \log\big(1 + (1+\alpha^2)\gamma\big) \ .$$

If we do not have the partial channel knowledge at the transmitter and send independent Gaussian signals with equal power from the two antennas, the average mutual information obtained is

$$\mathcal{I}_{\text{eq}} = \log\left(1 + \frac{1+\alpha^2}{2}\gamma\right) \ .$$

Thus the capacity, or spectral efficiency, gain with partial channel information in this case depends on the quality of the phase knowledge. At high SNRs, the spectral efficiency gain is an additive constant and has a range as

$$1 - \log(1+\alpha^2) \leq \mathcal{C}_{\text{gain}} \leq 1 \quad \text{(bps/Hz)} \ .$$

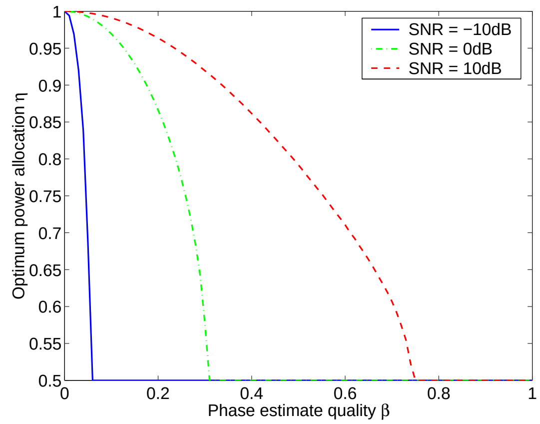
In terms of SNR advantage at the same transmission rate, the gain is

$$10\log_{10}(1+\alpha^2) \leq \text{SNR}_{\text{gain}} \leq 3 \ \ \text{(dB)} \ .$$

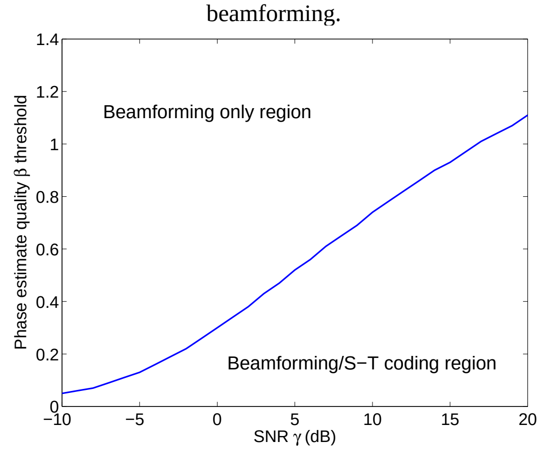
In Figure 7 we show the capacity and mutual information at various channel knowledge for both unequal and equal channel gain cases. The spectral efficiency gain depends on the channel gain ratio and the quality of the phase knowledge. The unequal channel gain plot illustrates that the knowledge of channel gain is valuable, even when the channel phase shift is unknown ($\beta = 0$). The power allocation to the two antennas can be regulated according to the known channel gain ratio to achieve a higher spectral efficiency than that obtained by independent and equal power transmission. For equal channel gain case, on the other hand, some phase knowledge is required to gain spectral efficiency over independent and equal power transmission from the two antennas.

VII. Conclusion

We have studied 2×1 MISO channels with partial transmit channel knowledge characterized by a known channel gain ratio and known phase shift PDF. The optimum signaling scheme is shown to be beamforming on the estimated channel phase shift with per antenna power adjusted according to the known parameters. While in the analysis we assume that the channel

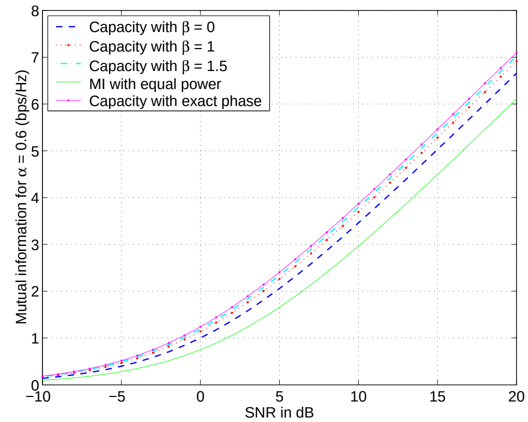


(a) Portion of the total power allocated to the first antenna in beamforming.

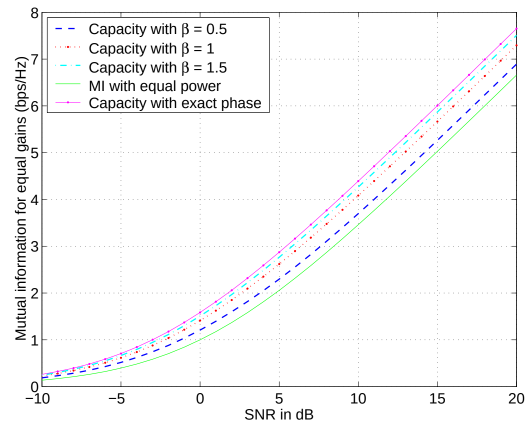


(b) Phase estimate quality threshold above which only beamforming is optimum.

Fig. 6. Equal channel gain case.



(a) Unequal channel gain with $\alpha = 0.6$.



(b) Equal channel gain.

Fig. 7. Capacity and mutual information at various channel knowledge.

coefficients have fixed amplitudes, the same results also apply to channels with high K factors, where the amplitudes of the channel coefficients can be estimated and is known, but the channel phase is unknown and random. In such situations, simple beamforming scheme is optimum from the ergodic capacity point of view. Partial channel knowledge therefore can help to simplify the transmission scheme significantly, and at the same time provides a spectral efficiency gain of up to 1 bps/Hz or a SNR advantage of up to 3dB using two transmit antennas.

REFERENCES

[1] T.M.Cover and J.A.Thomas, *"Elements of Information Theory"*, John Wiley & Sons, Inc., 1991.

[2] A.Paulraj, R.Nabar and D.Gore, *"Introduction to Space-Time Wireless Communications"*, Cambridge University Press, May 2003.

[3] D.S.Baum, et. al., *"Measurement and Characterization of Broadband MIMO Fixed Wireless Channels at 2.5GHz"*, Int. Conf. Per. Wireless Comm., pp.203-206, Dec 2000.

[4] S.Boyd and L.Vandenberghe, *"Convex Optimization"*, Cambridge University Press 2003. Also avail. at URL http://www.stanford.edu/ boyd/cvxbook.html.

[5] I.E.Telatar, *"Capacity of Multi-antenna Gaussian Channels"*, AT&T-Bell Lab. Int. Tech. Mem., June 1995.

[6] E.Visotsky and U.Madhow, *"Space-Time Transmit Precoding with Imperfect Feedback"*, IEEE Trans. on Info. Theory, pp. 2632-2639, Vol. 47, No. 6, Sep. 2001.

[7] A.Gorokhov, *"Capacity of Multiple-Antenna Rayleigh Channel with a Limited Transmit Diversity"*, ISIT 2000, pp. 411, Sorrento, Italy, June 2000.

[8] S.Jafar and A.Goldsmith, *"Transmitter Optimization and Optimality of Beamforming for Multiple Antenna Systems"*, to appear in IEEE Tran. Wireless Comm.

[9] S.Venkatesan, S.H.Simon and R.A.Venezuela, *Capacity of a Gaussian MIMO channel with non-zero mean*, Proc. VTC, Florida, Oct 2003.

[10] M.Vu and A.Paulraj, *Some asymptotic capacity results for MIMO wireless with and without channel knowledge at the transmitter*, Proc. 37th Asilomar Conf. Sig., Sys. and Comp., pp. 258-262, Nov. 2003.

[11] H.Sampath and A.Paulraj, *"Linear Precoding for Space-Time Coded Systems with Known Fading Correlations"*, IEEE Comm. Lett., Vol. 6, pp. 239-241, June 2002.

[12] G.Jongren, M.Skoglund and B.Ottersten, *"Combining Beamforming and Orthogonal Space-Time Block Coding"*, IEEE Trans. on Info. Theory, Vol 48, pp. 611-627, Mar 2002.

[13] M.Vu and A.Paulraj, *"Linear Precoding for MIMO Channels with Non-zero Mean and Transmit Correlation in Orthogonal Space-Time Coded Systems"*, to appear in Proc. VTC, Sept 2004.

[14] S.O.Rice, *"Statistical Properties of a Sine Wave plus Random Noise"*, Bell Sys. Tech. J. 27, pp. 109-157, Jan 1948.

[15] P. Beckmann, *"Probability in Communication Engineering"*, Harcourt, Brace and World, 1967.

[16] W.C.Jakes, ed., *"Microwave Mobile Communications"*, IEEE Press, 1993.

[17] P.Z.Peebles,JR., *"Probability, Random Variables, and Random Signal Principles"*, McGrawHill, 1993.